Komplex függvények színes ábrázolása

Lócsi Levente

ELTE IK

2006. december

Tartalom

Miről lesz szó?

- $\mathbb{C} \to \mathbb{C}$ függvények
- Különféle ábrázolási módok
- Színes ábrázolás
- Programok
- Néhány konkrét függvény

$\mathbb{C} \to \mathbb{C}$ *függvényekről*

- A komplex számokat egy síkon képzelhetjük el
- Valós rész, képzetes rész
- Komplex függvények ábrázolásának nehézsége

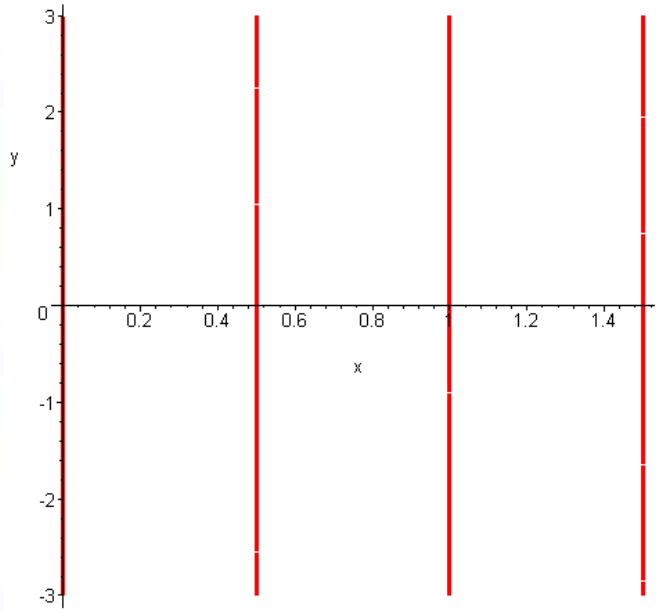
„Life is complex. It has real and imaginary components.”

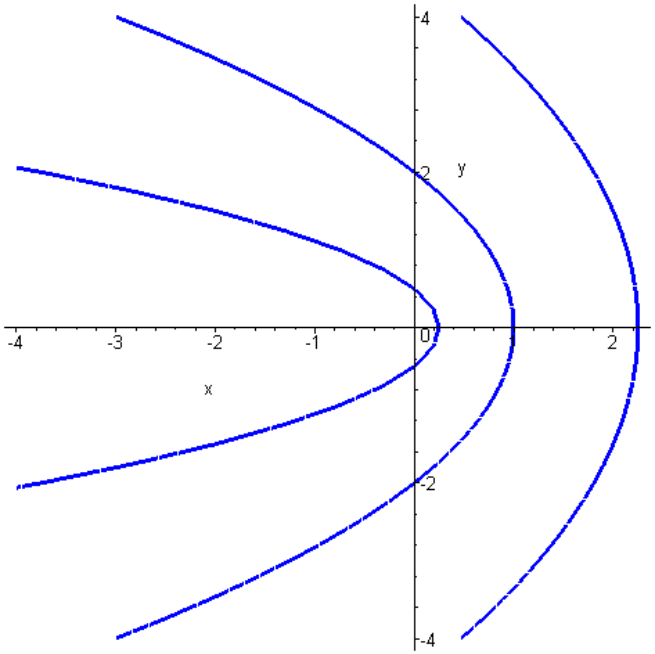
Különféle ábrázolási módok

Ábrázolási módok – 1

Két síkos (z és w) ábrázolás

Pl.: $f(z) = z^2$

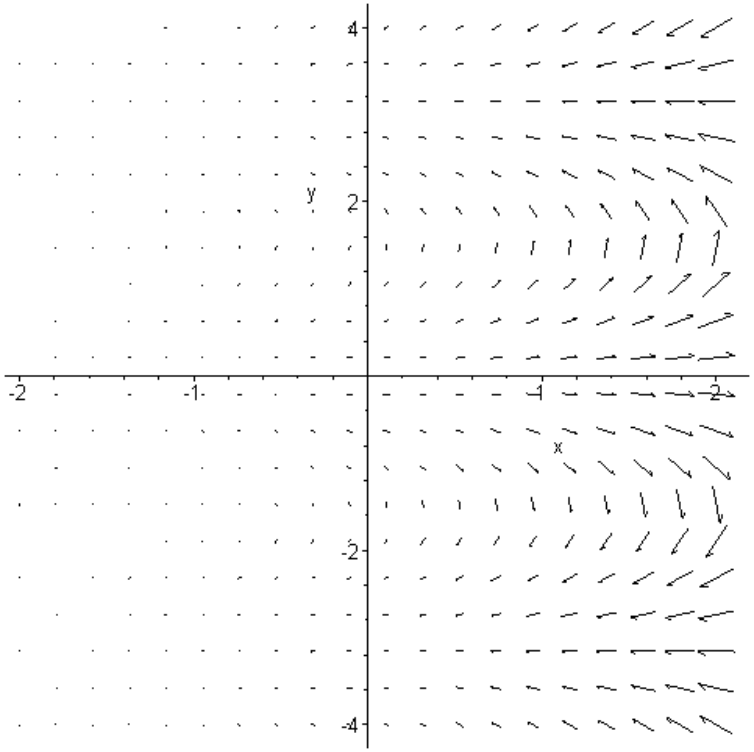




Ábrázolási módok – 2

Vektoros, erőterek szemléltetéséhez hasonló

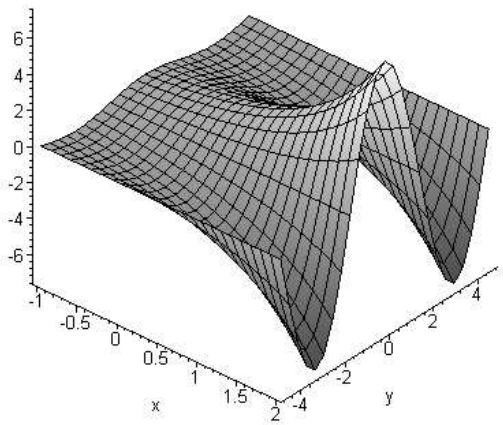
Pl.: $f(z) = \exp z$



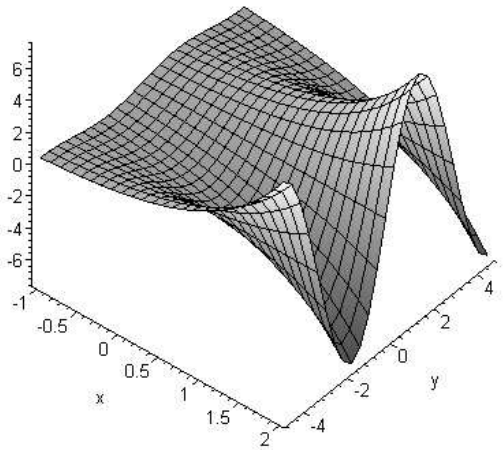
Ábrázolási módok – 3.1

3D, külön valós ill. képzetes rész. $\mathbb{R}^2 \to \mathbb{R}$

Pl.: $f(z) = \exp z$



$f_1(x, y) = e^x \cos y$

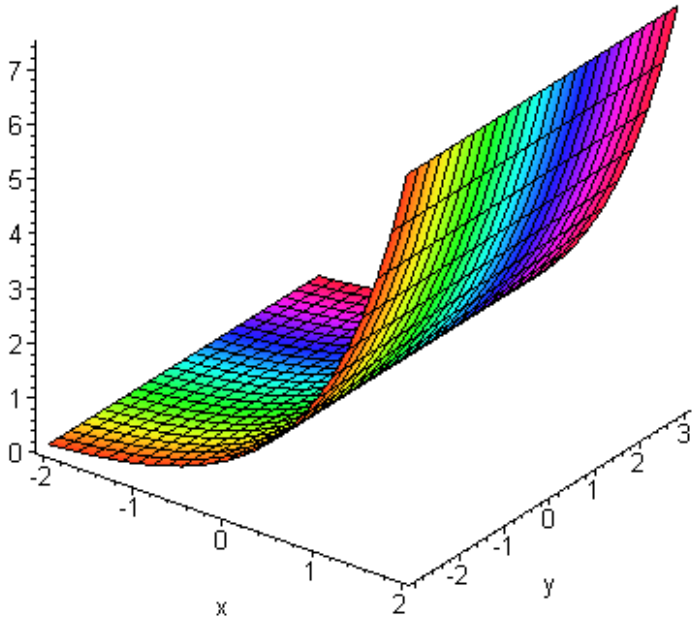


$f_2(x, y) = e^x \sin y$

Ábrázolási módok – 3.2

Még mindig 3D + szín

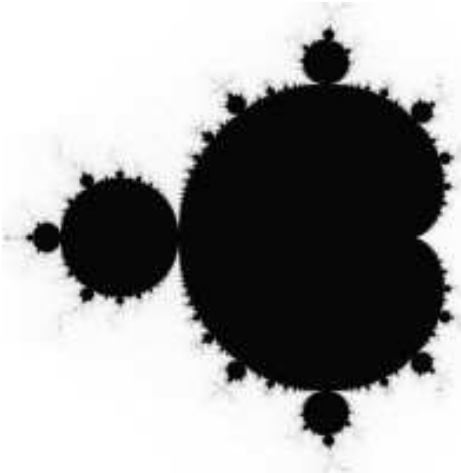
Pl.: $f(z) = \exp z$



Ábrázolási módok – 4

Fraktál színezések

Pl.: *Mandelbrot*



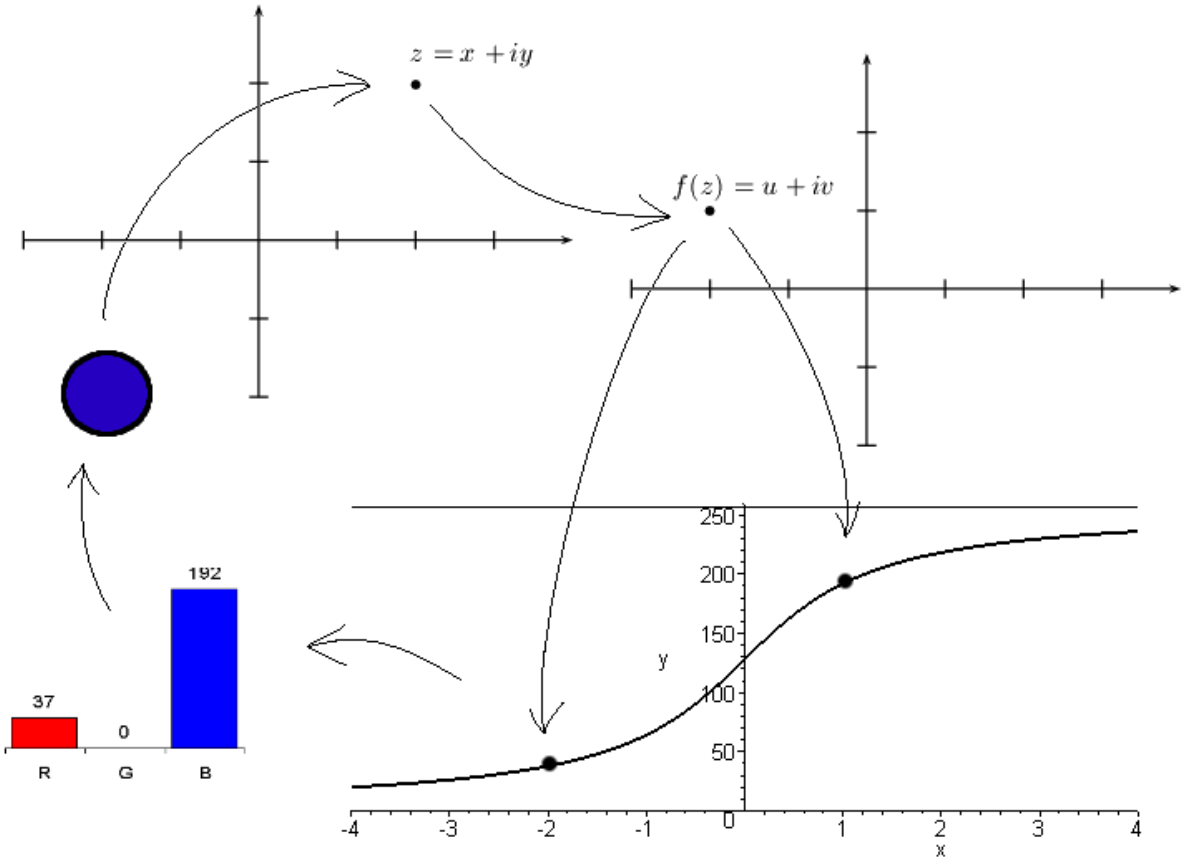
Színes ábrázolás

Színes ábrázolás elve

- Komplex számsík pontjaihoz színek $\mathbb{C} \to \mathbb{B}^3$, ahol $\mathbb{B} = [0..255]$, RGB
- Egy síkon a pontokat a képpontnak ($\in \mathbb{C}$) megfelelő színre színezzük
- Többféle *színezés*

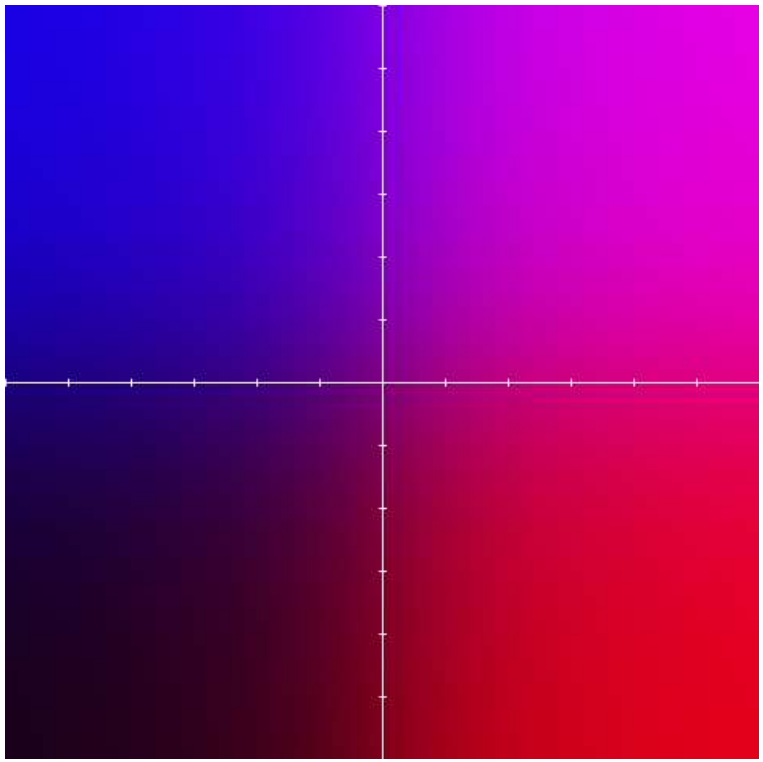
ImRe színezés

- Külön valós, képzetes részt. ($\mathbb{R} \to \mathbb{B}$)



ImRe színezés

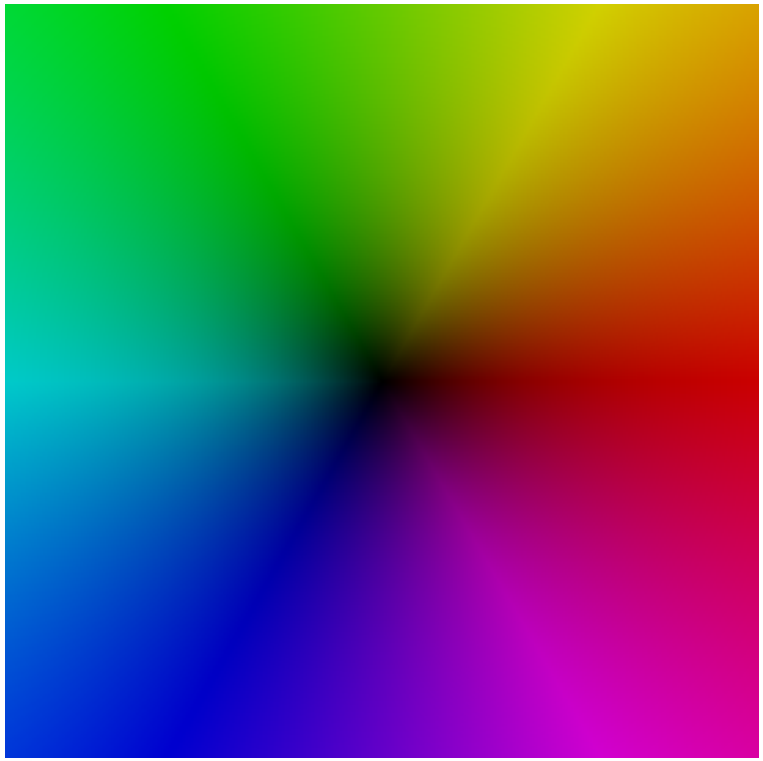
- Külön valós, képzetes részt. ($\mathbb{R} \to \mathbb{B}$)



$f(z) = z$

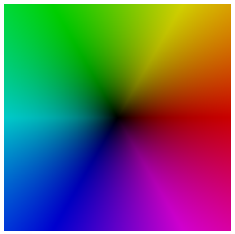
Arg + Abs

- Egy szivárványt rátekerünk az origóra. . . + sötétítünk



Arg + Abs

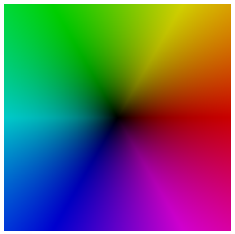
- Egy szivárványt rátekerünk az origóra. . . + sötétítünk



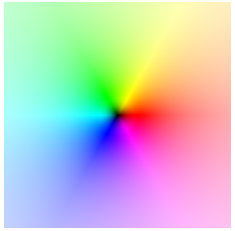
- + világosodás

Arg + Abs

- Egy szivárványt rátekerünk az origóra. . . + sötétítünk



- + világosodás



- $\exists$ más mód. . . , + variációk
  - csak abszolút érték
  - sötét – világos irány, lépcsősítés

Programok

Standard C++

Váljon egyszerűvé ilyen képek készítése…

Objektumorientált szemlélet

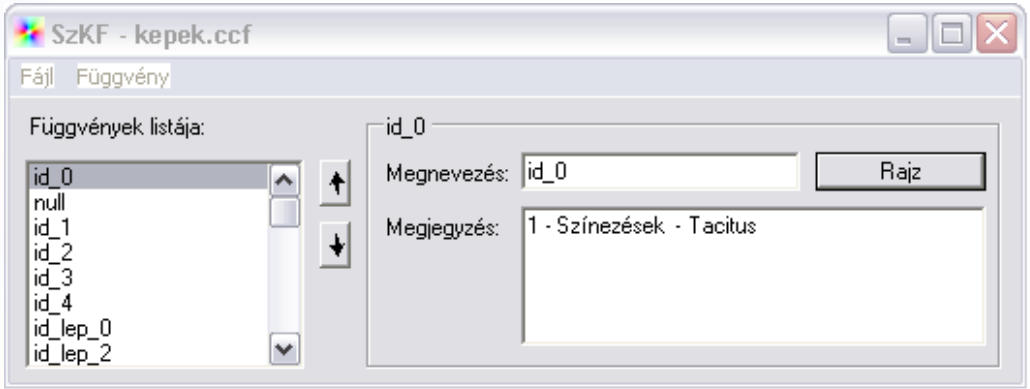
- `class KomplAbra {...};`
- `class IdSzamolo : public Szamolo {...};`
- `class ArgKorSzinezo : public Szinezo {...};`

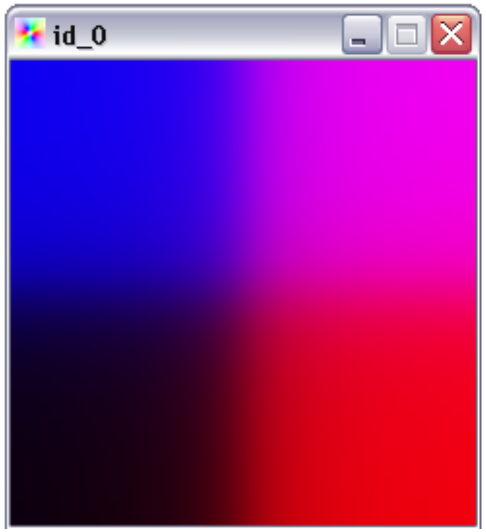
Egyszerű példakód

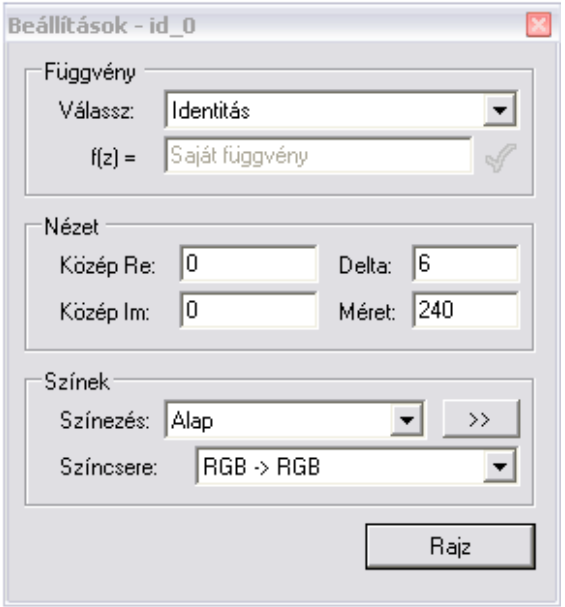
```
#include "abra.h"

int main()
{
    KomplAbra* a = new KomplAbra(0,0,4,480);
    ExpSzamolo* e = new ExpSzamolo();
    AlapSzinezo* sz = new AlapSzinezo();

    a->SetFuggveny(e);
    a->SetSzinezes(sz);
    a->SetTengely(true);

    a->CreateAbra();
    a->KiirBMP("exp.bmp");

    return 0;
}
```

Színes Komplex Függvények

Microsoft Visual Studio .NET 2003







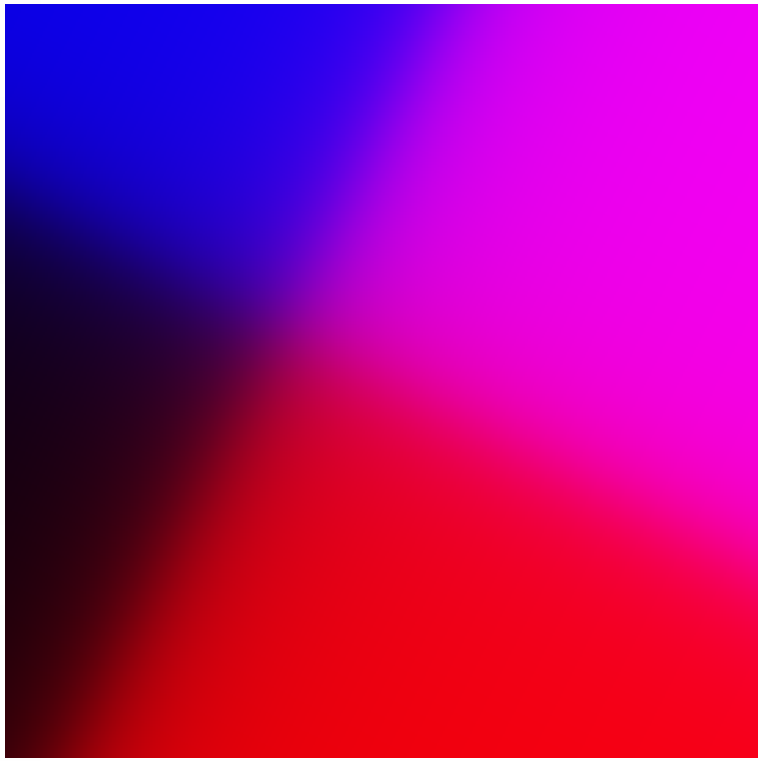
Néhány függvény

Konjugált

- $f(z) = \overline{z}$

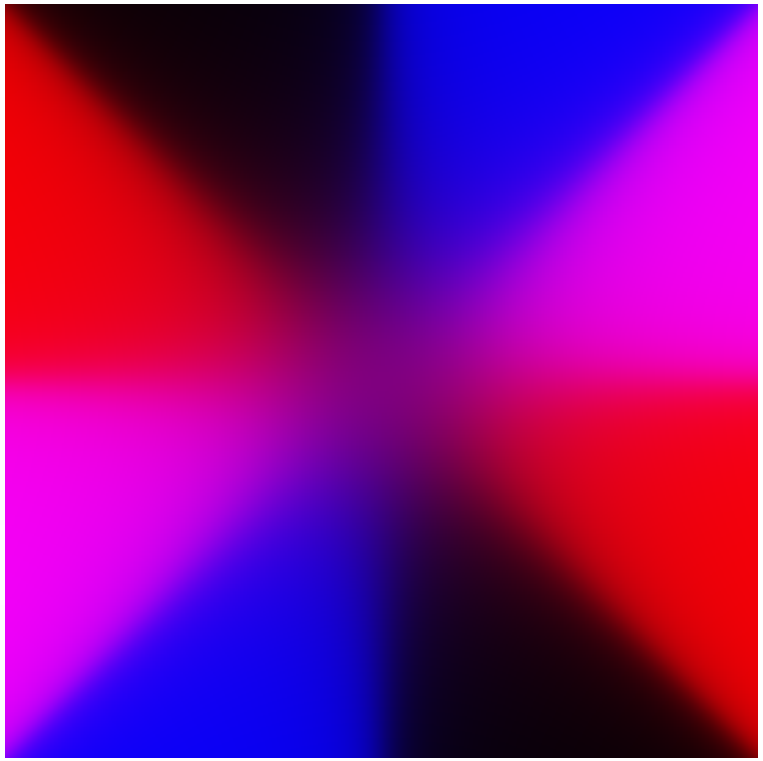
Lineáris

- $f(z) = (2+i)z + 2$



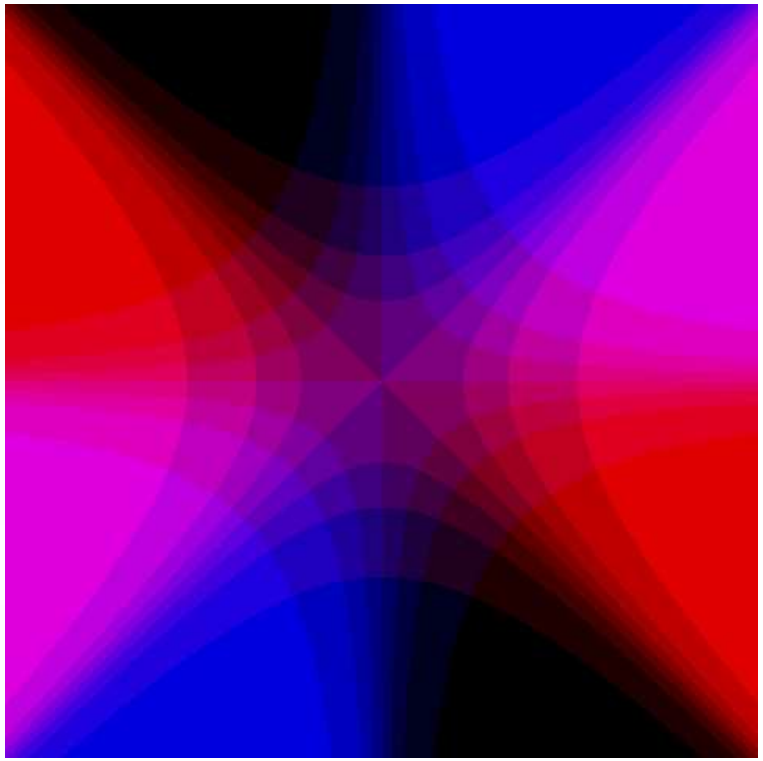
Négyzet – 1

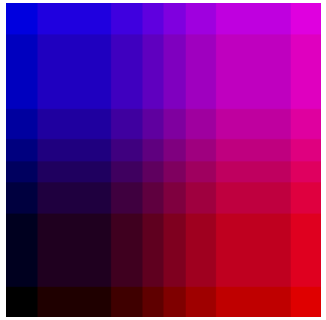
- $f(z) = z^2$



Négyzet – 2

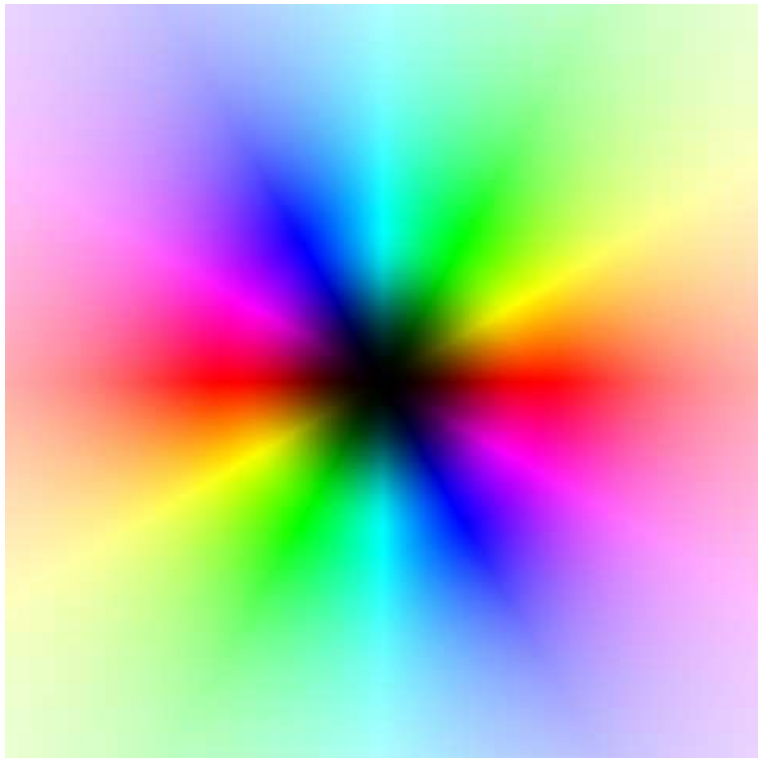
- $f(z) = z^2$

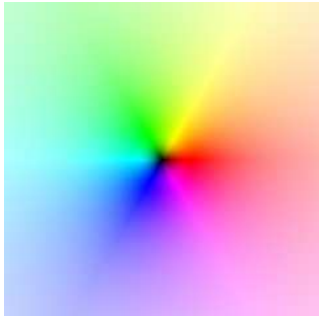




Négyzet – 3

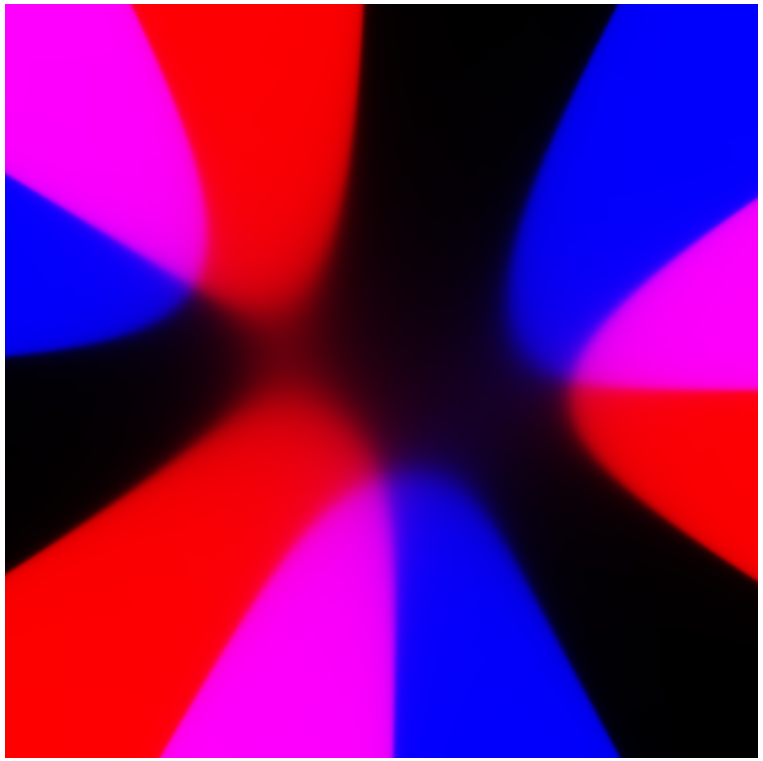
- $f(z) = z^2$





Polinom

- $f(z) = (z-2)(z+i)(z+2-i)$

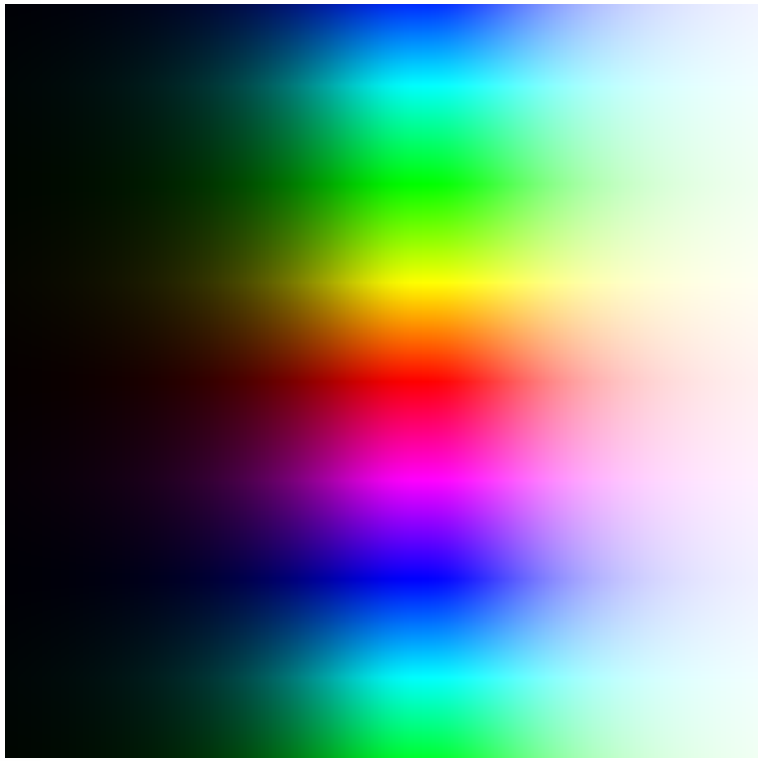


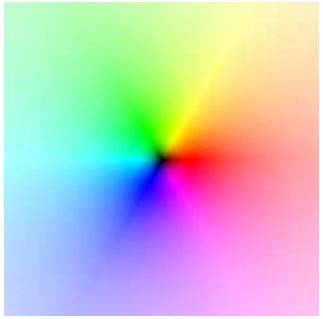
Exponenciális – 1

- $f(z) = \exp z$

Exponenciális – 2

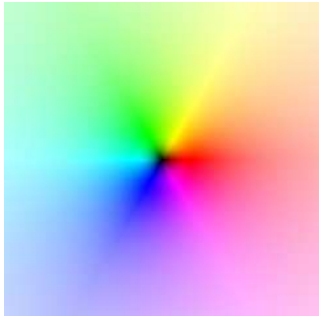
- $f(z) = \exp z$





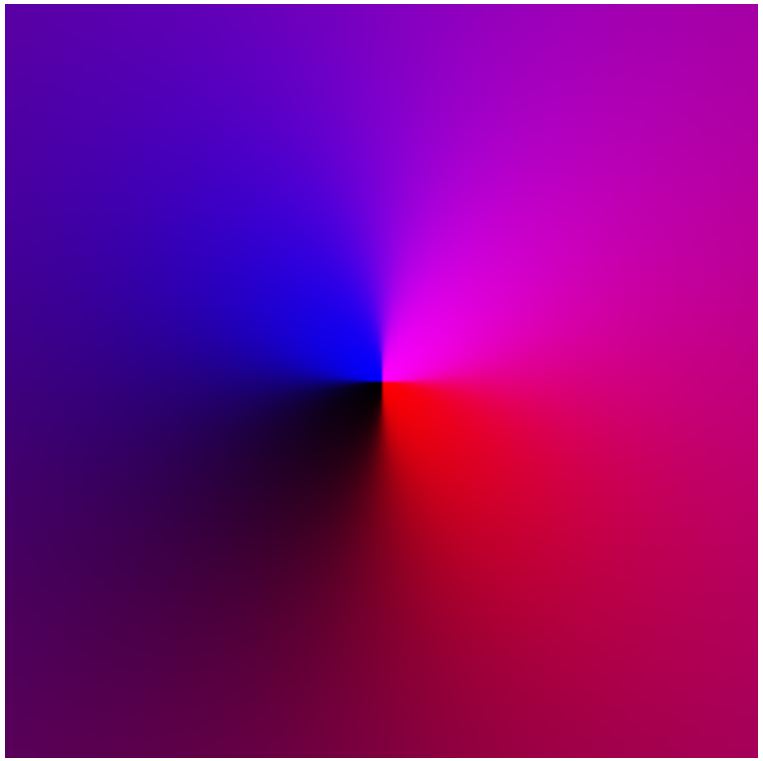
Logaritmus

- $f(z) = \log z$ *(főág)*



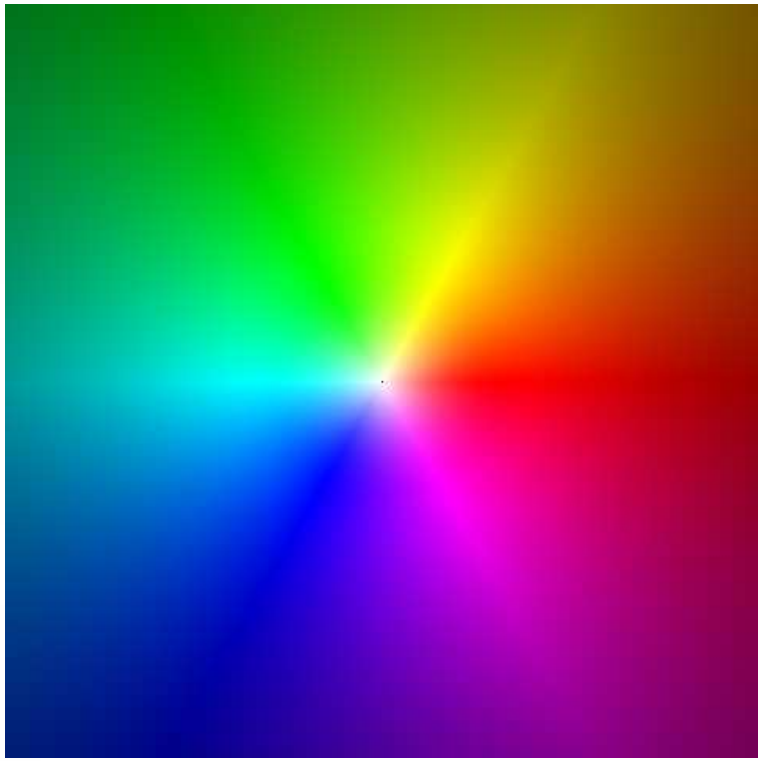
Inverzió – 1

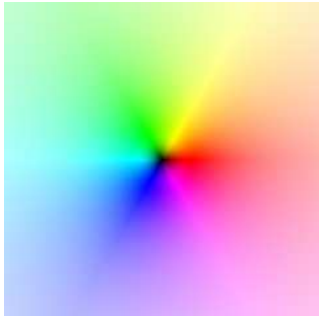
- $f(z) = 1/\overline{z}$



Inverzió – 2

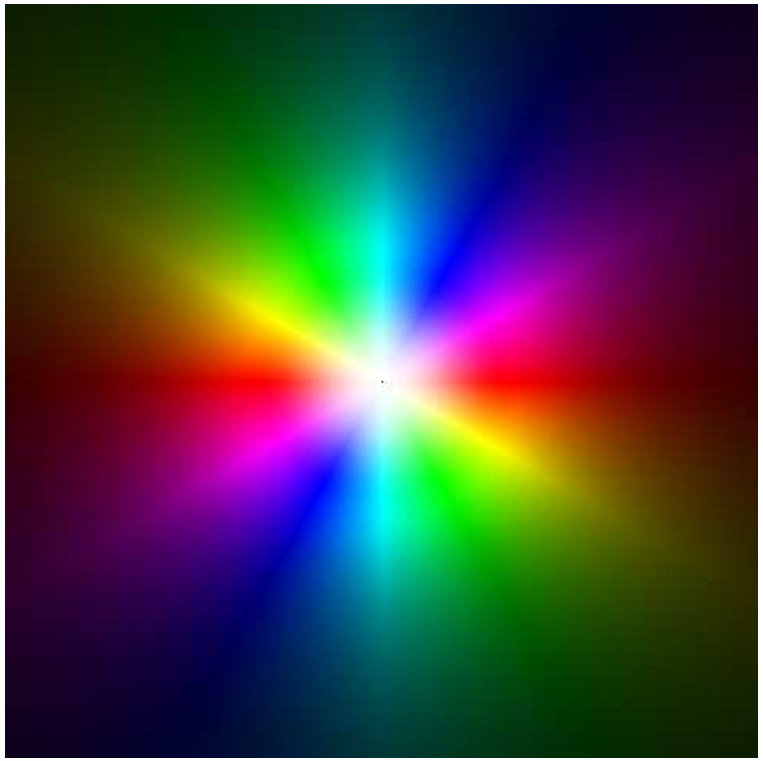
- $f(z) = 1/\overline{z}$

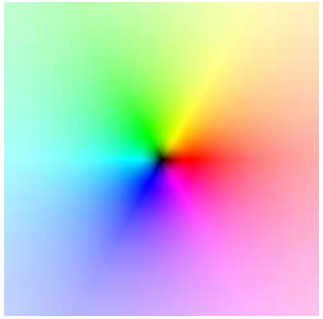




Egy másodrendű pólus

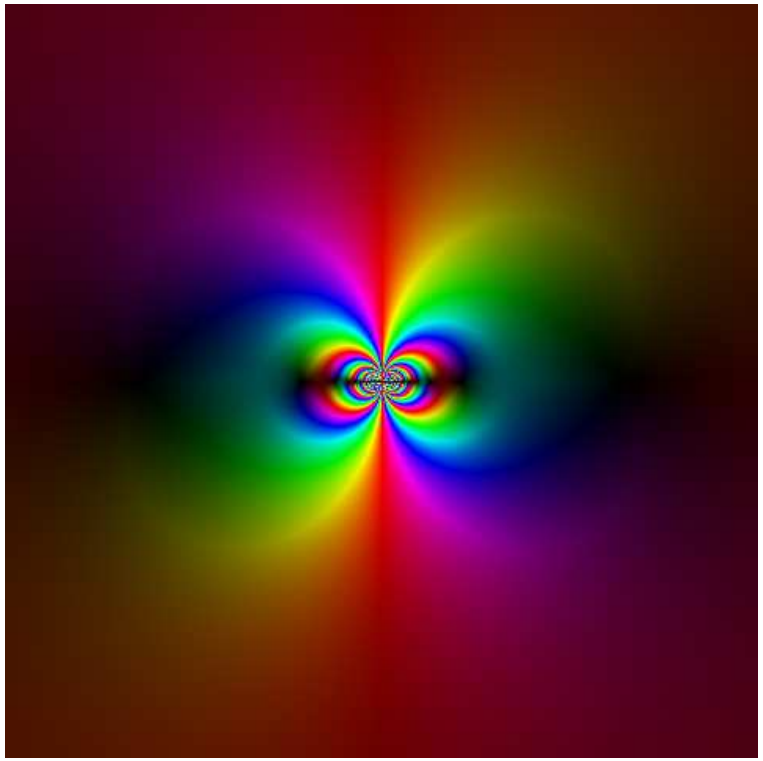
- $f(z) = 1/z^2$

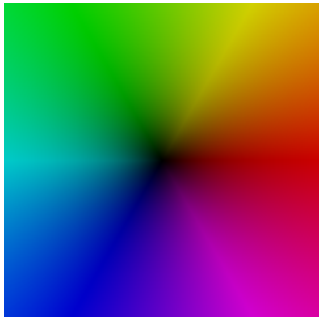




Egy lényeges szingularitás

- $f(z) = \cos \frac{1}{z}$





Hatványsorok – exp

- $$T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$$

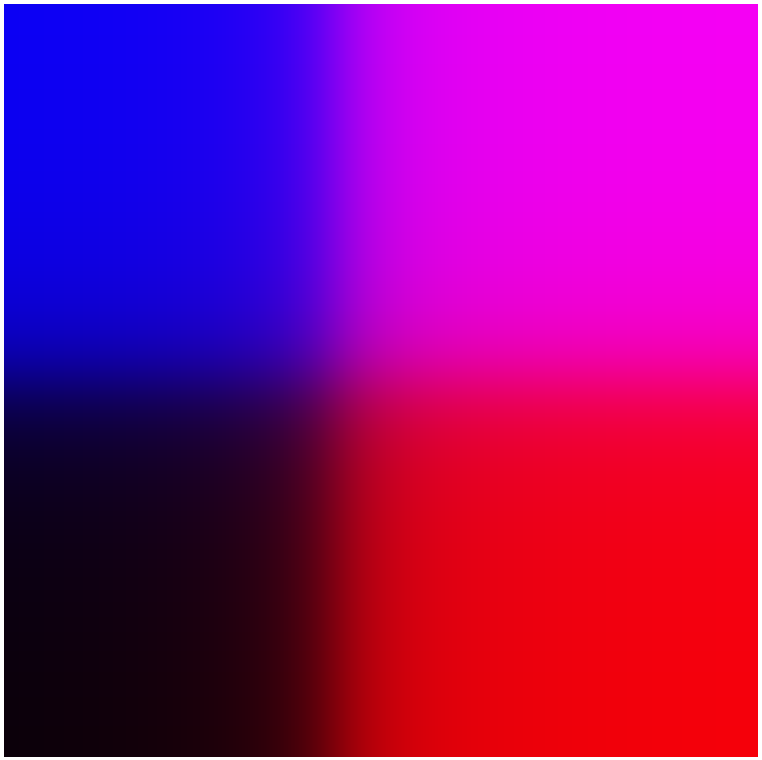
Hatványsorok – exp

- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$

$n = 0$

Hatványsorok – exp

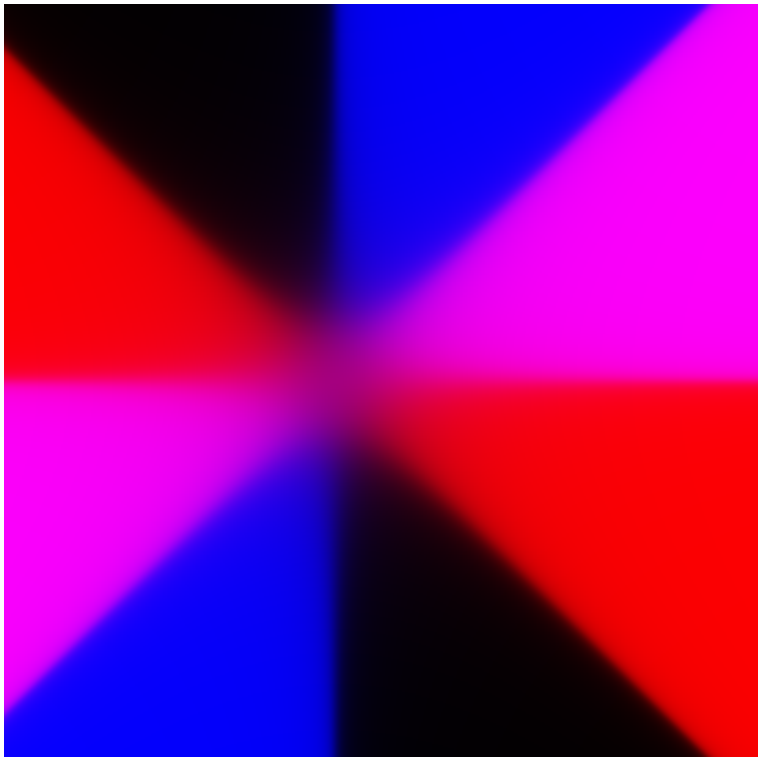
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 1$

Hatványsorok – exp

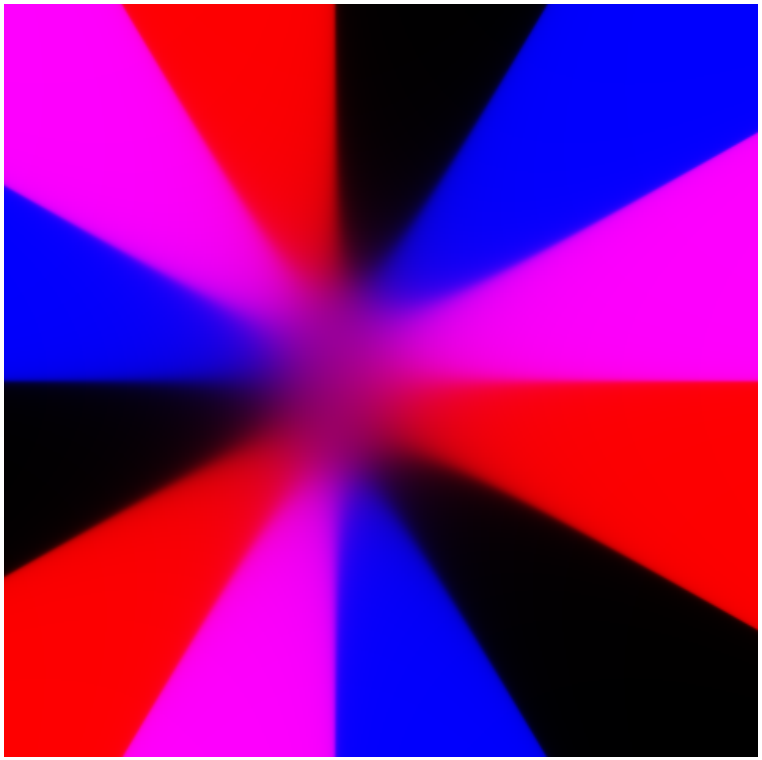
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 2$

Hatványsorok – exp

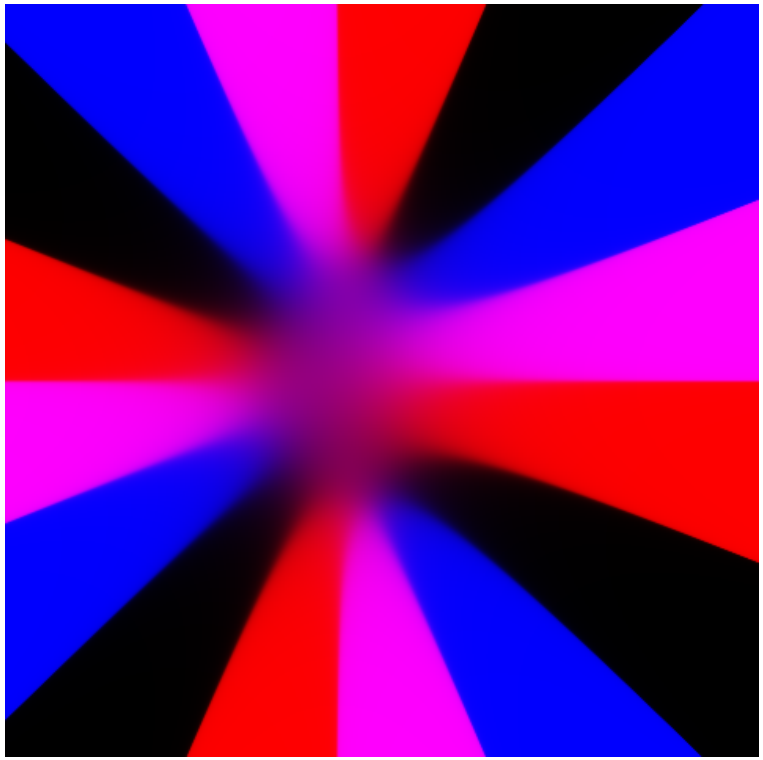
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 3$

Hatványsorok – exp

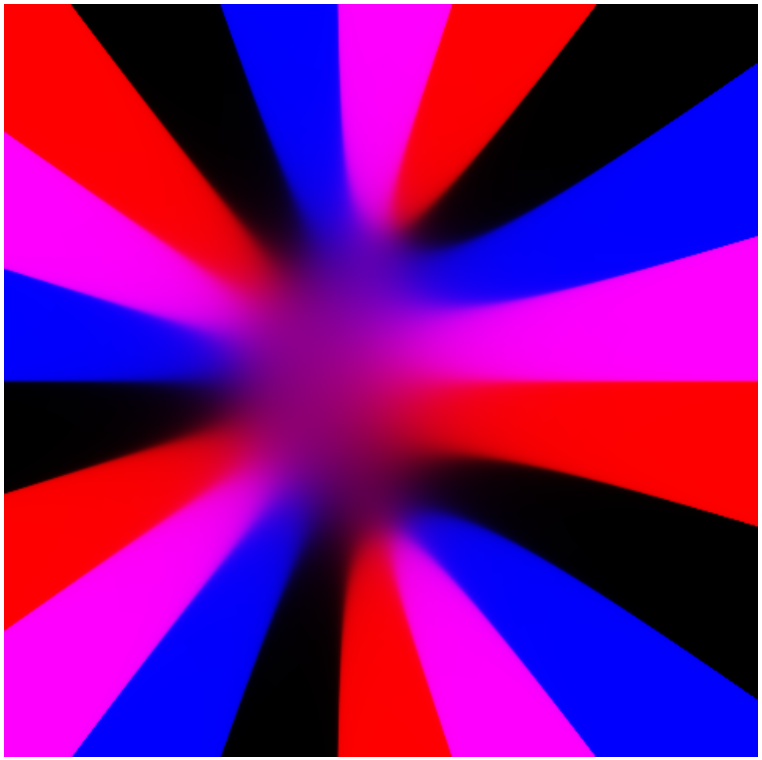
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 4$

Hatványsorok – exp

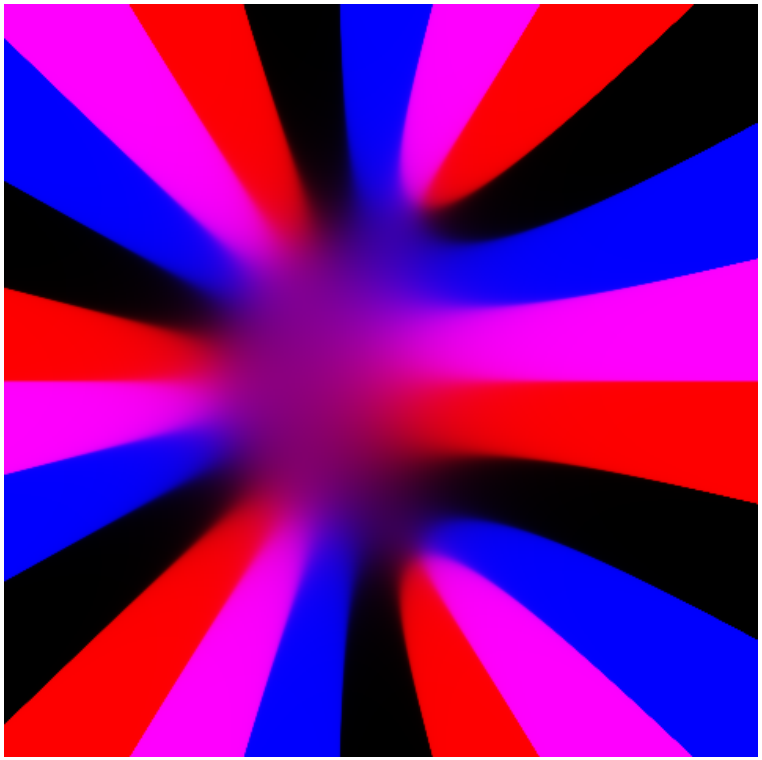
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 5$

Hatványsorok – exp

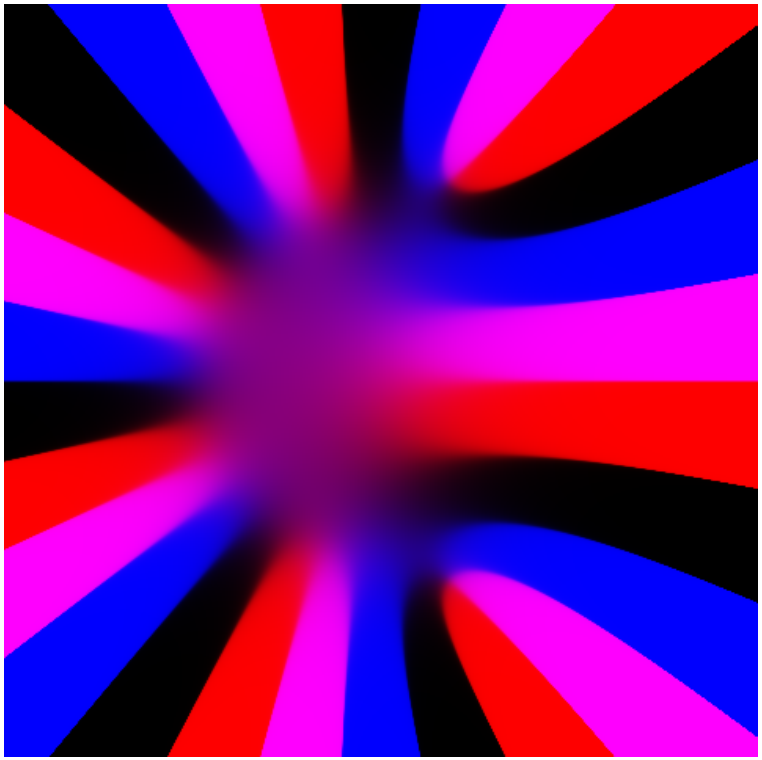
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 6$

Hatványsorok – exp

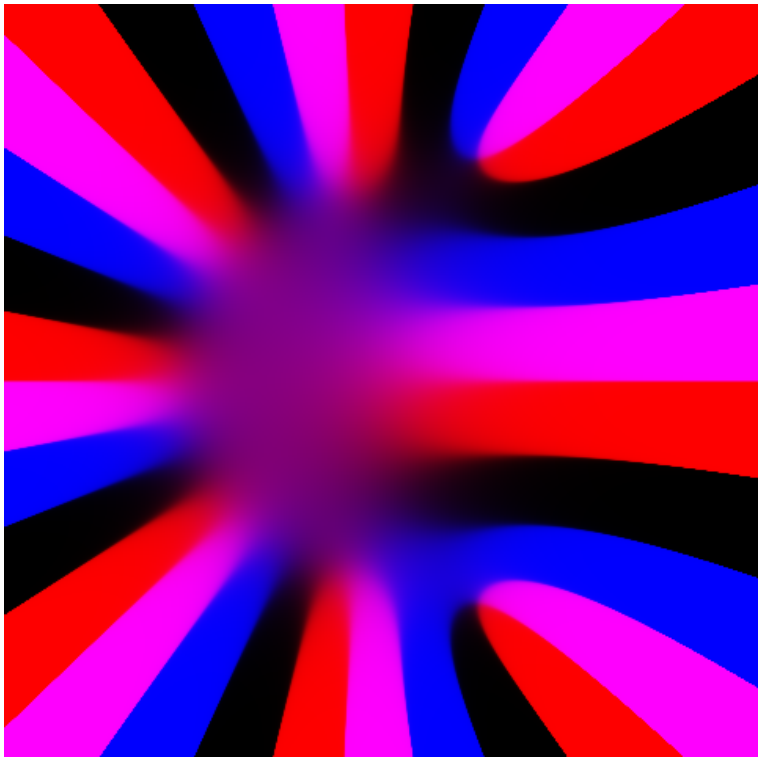
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 7$

Hatványsorok – exp

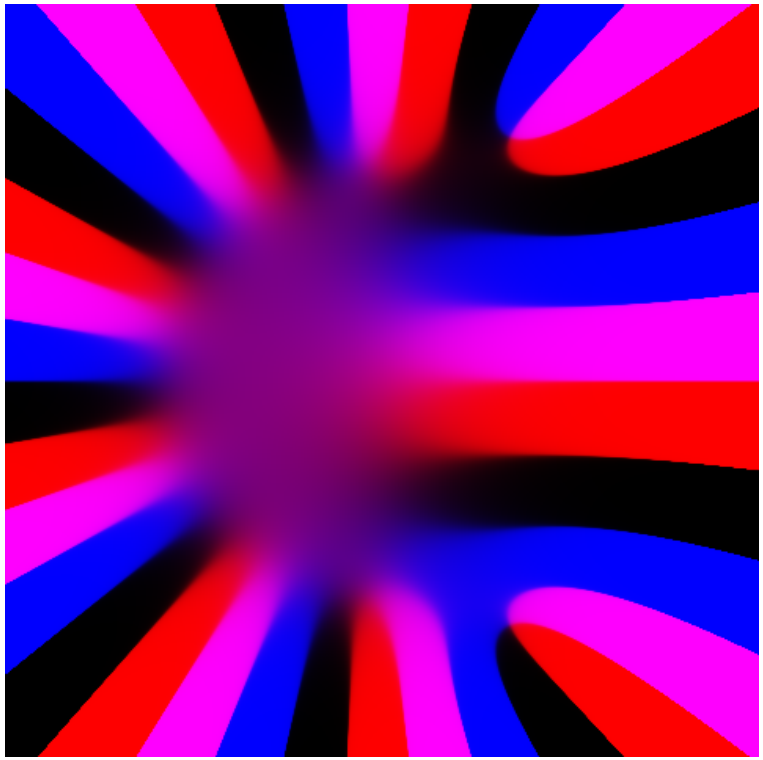
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 8$

Hatványsorok – exp

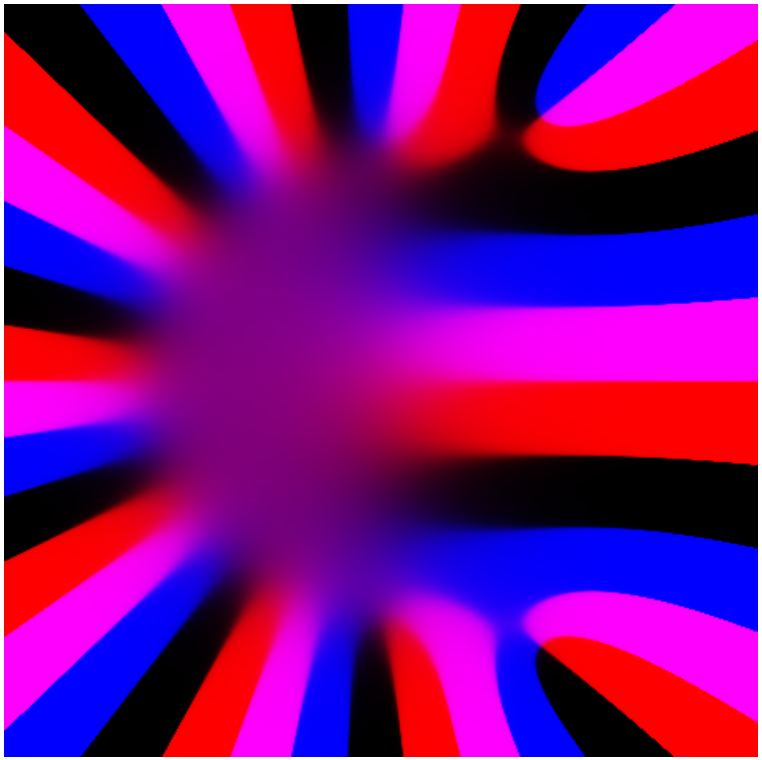
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 9$

Hatványsorok – exp

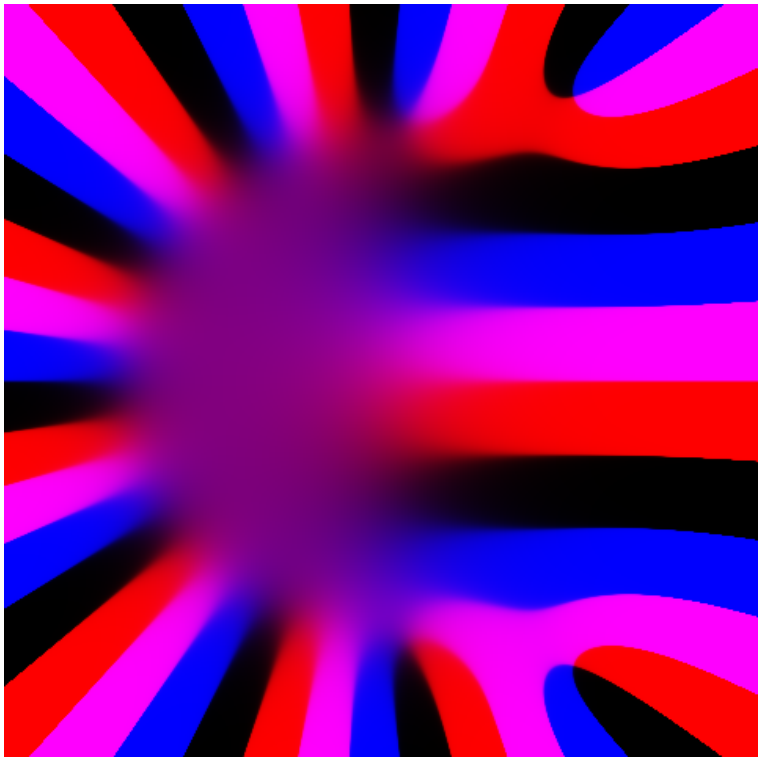
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 10$

Hatványsorok – exp

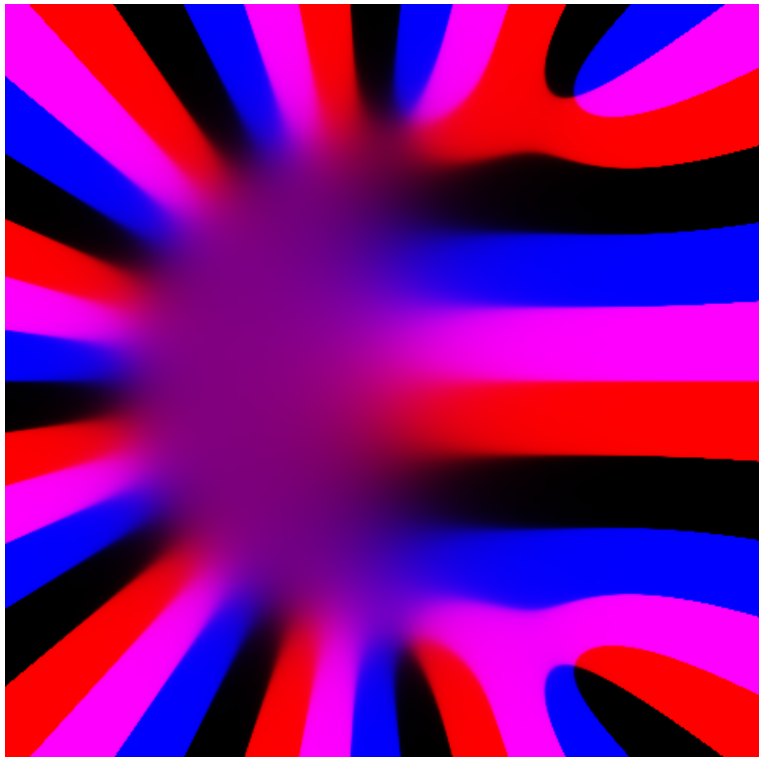
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$



$n = 11$

Hatványsorok – exp

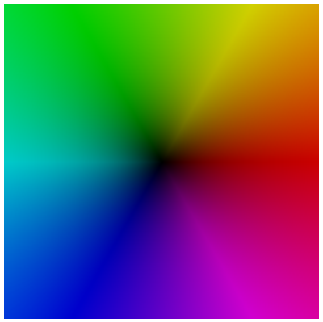
- $T_n(z) = \sum_{k=0}^{n} \frac{z^n}{n!}$

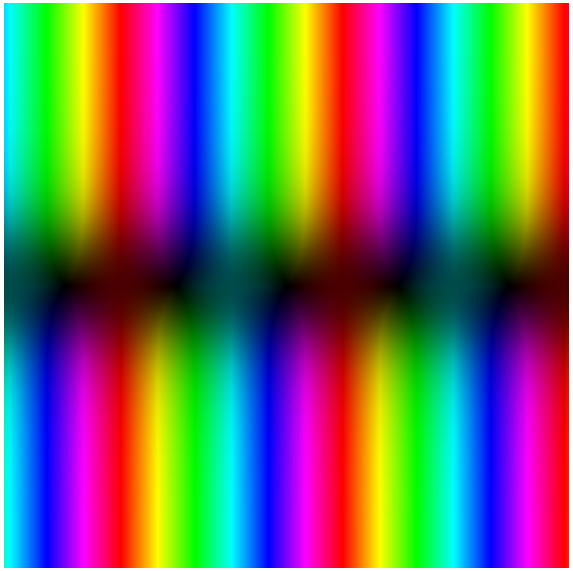


$n = 11$ Ennyi.

Hatványsorok – sin

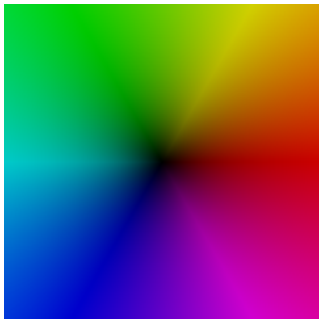
- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$

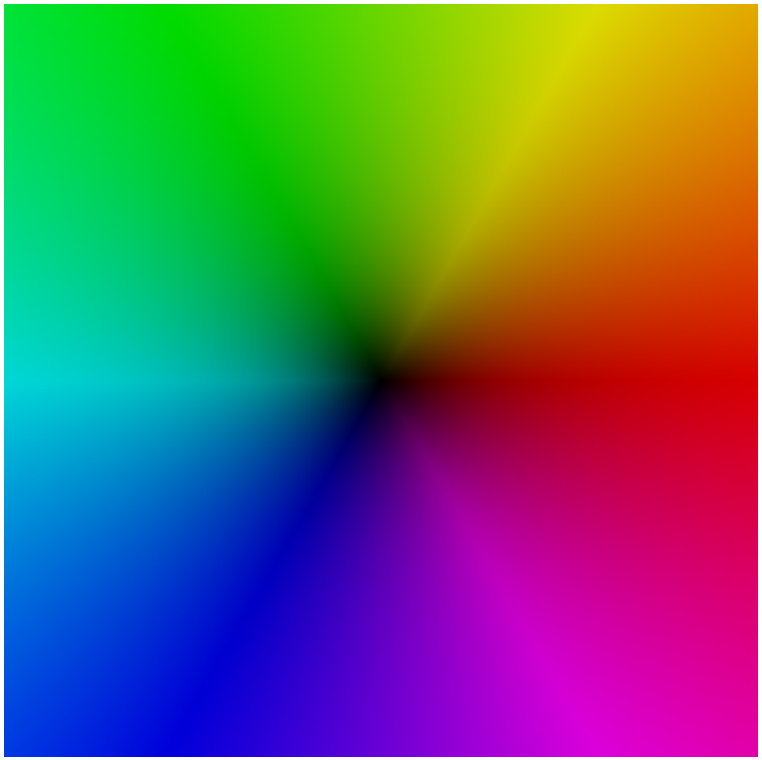


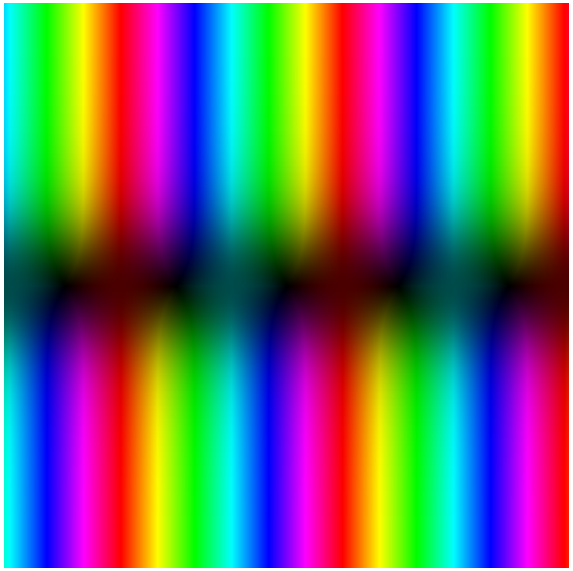


Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$



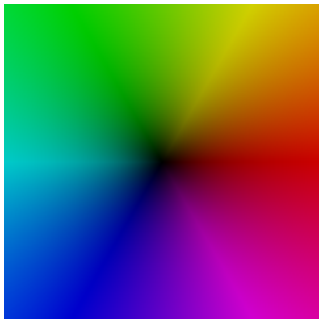


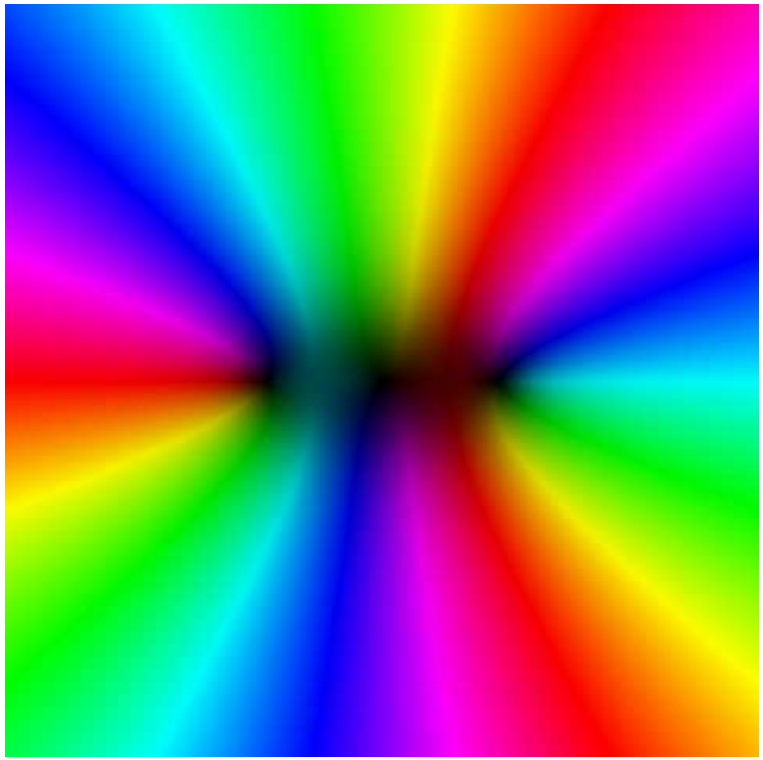


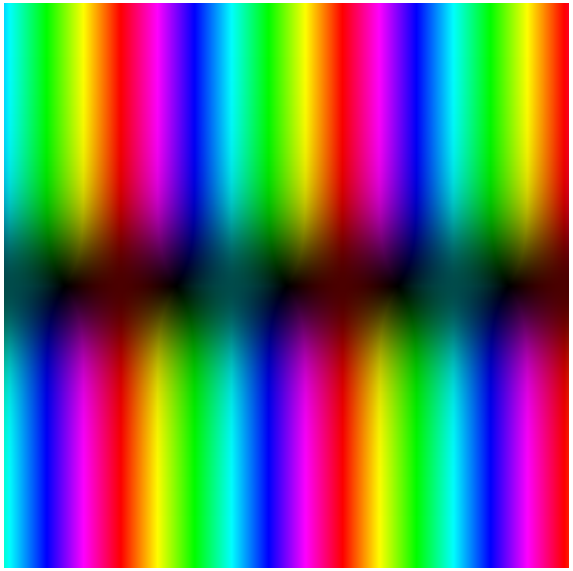
$n = 0$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$



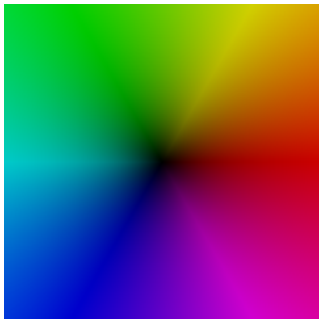


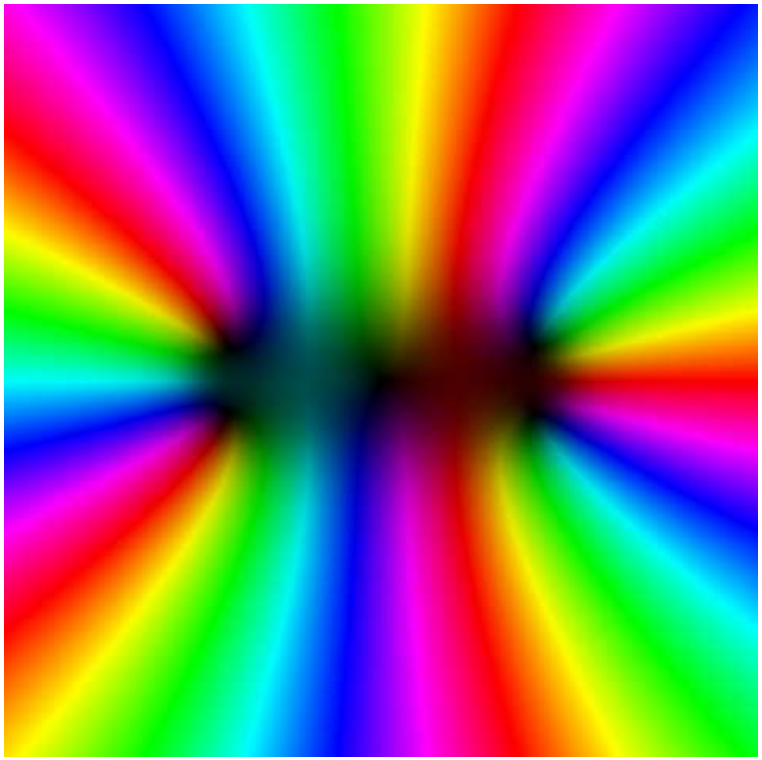


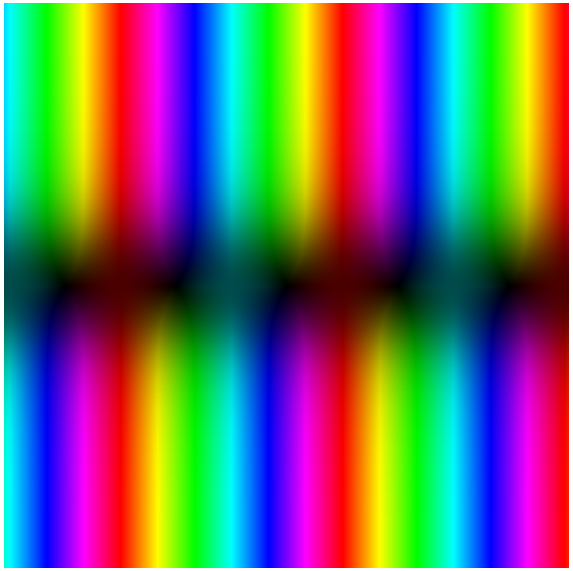
$n = 1$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$



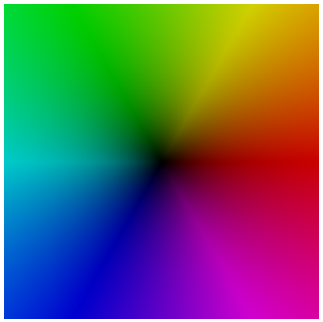


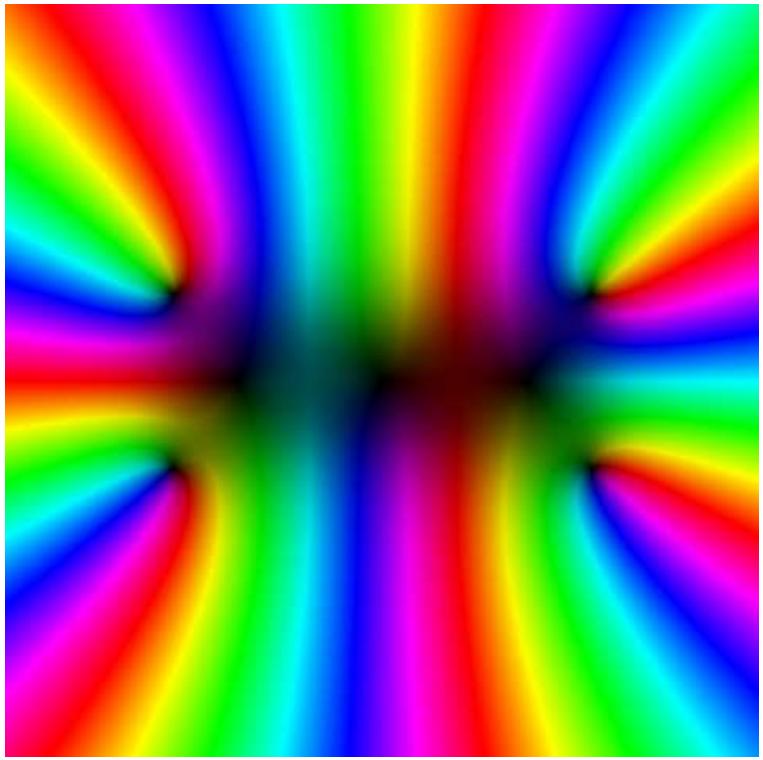


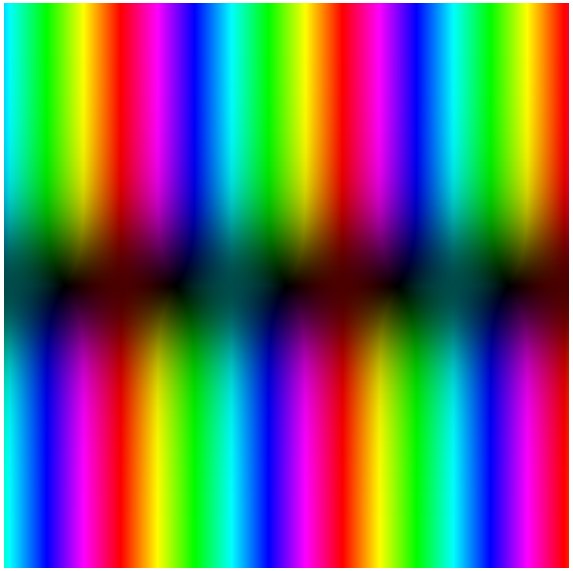
$n = 2$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$



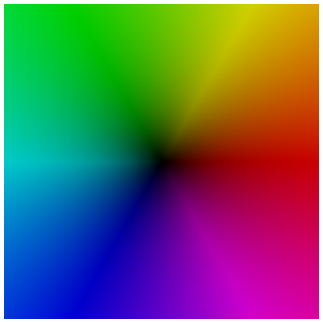


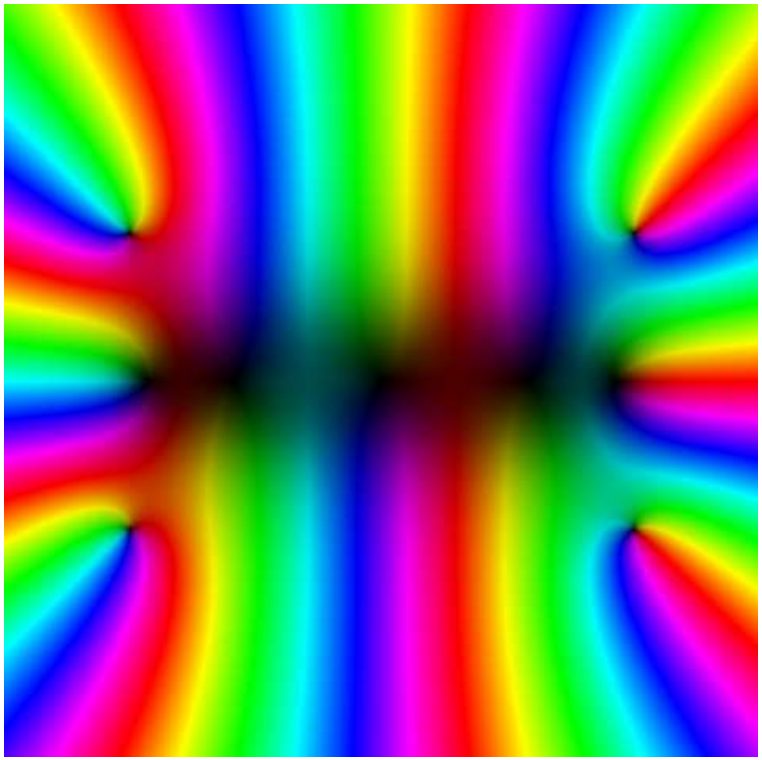


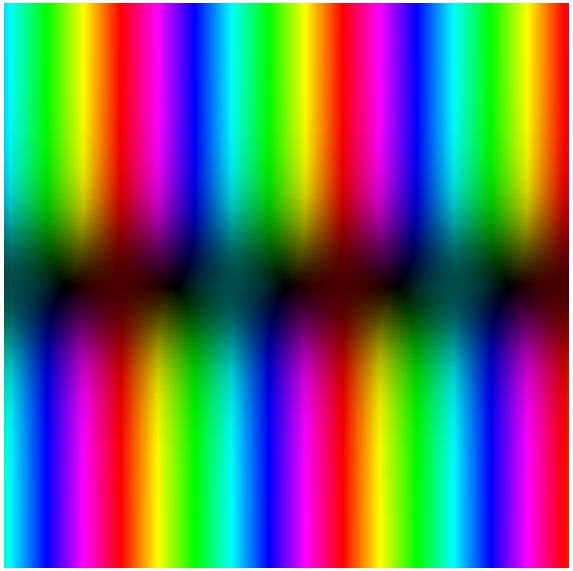
$n = 3$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$



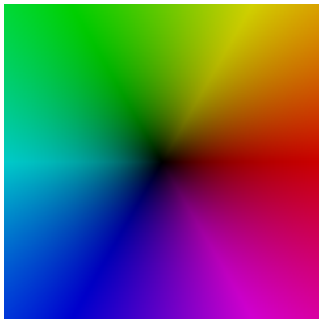


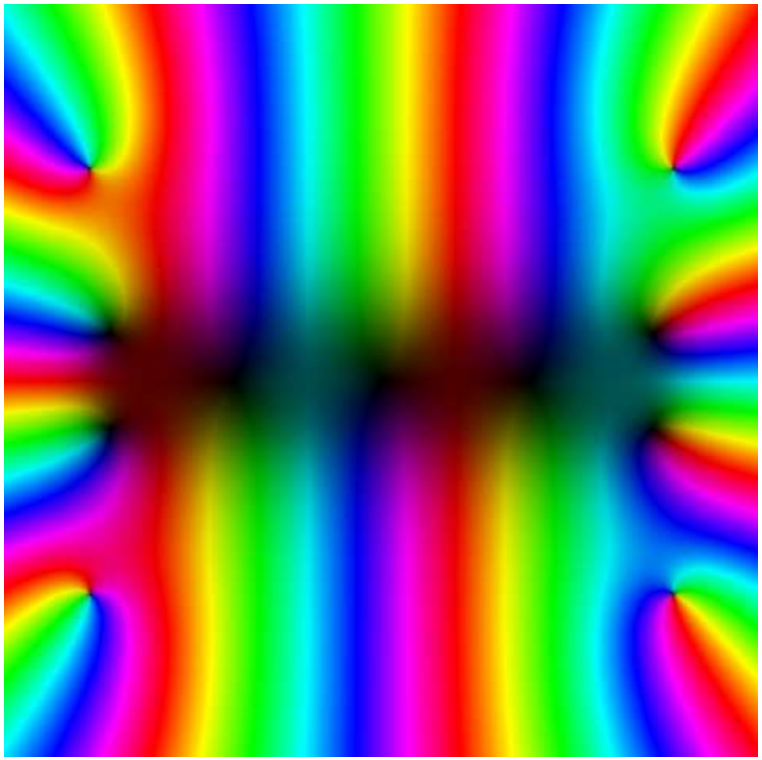


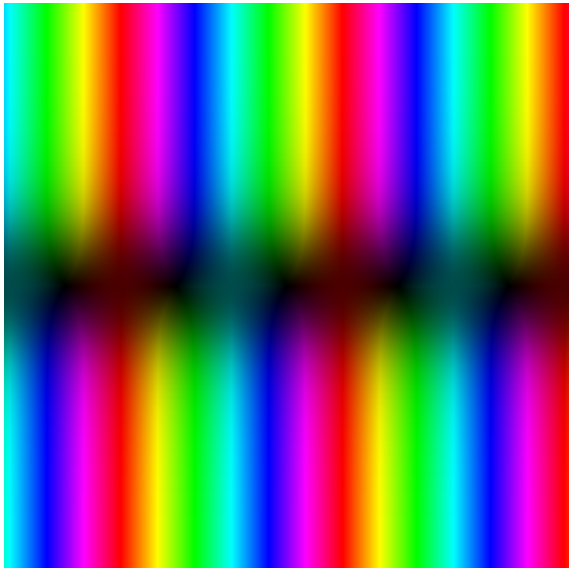
$n = 4$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$



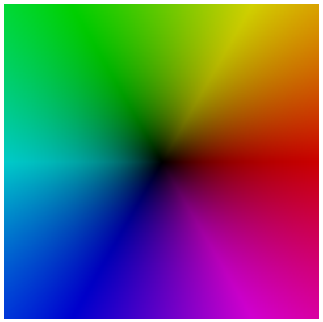


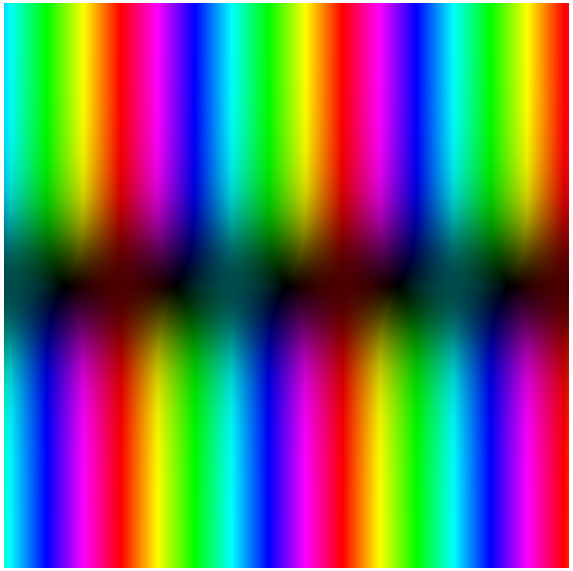


$n = 5$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$

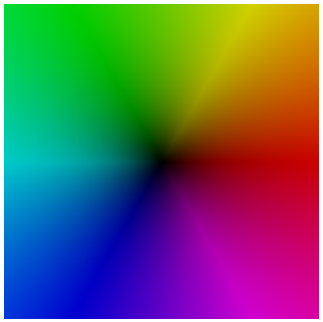


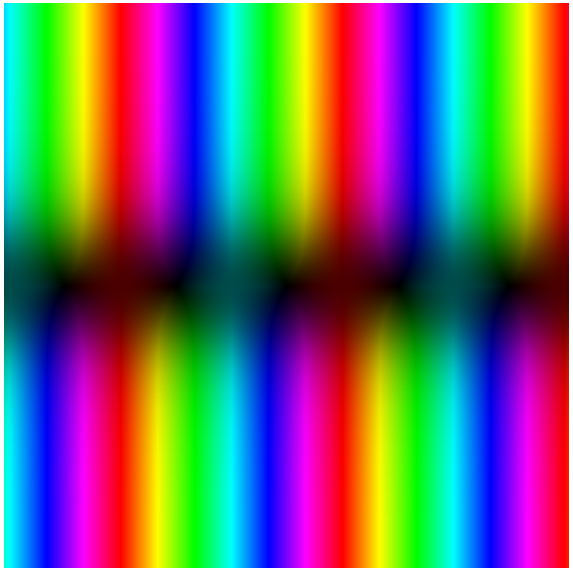


$n = 6$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$

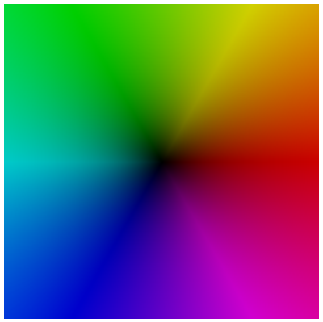


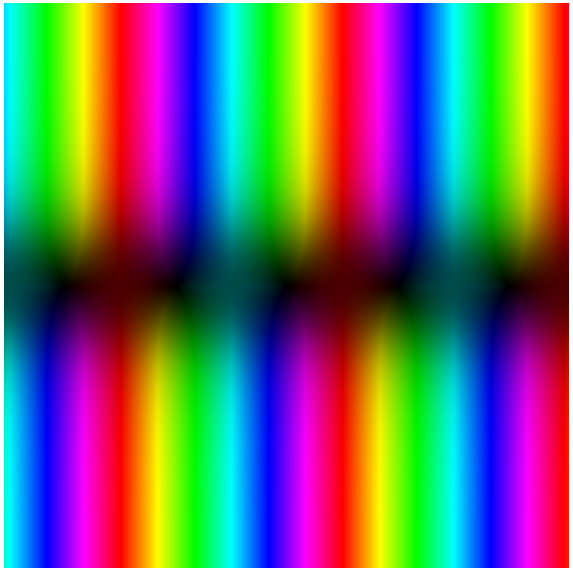


$n = 7$

Hatványsorok – sin

- $T_n(z) = \sum_{k=0}^{n} (-1)^n \frac{z^{(2n+1)}}{(2n+1)!}$

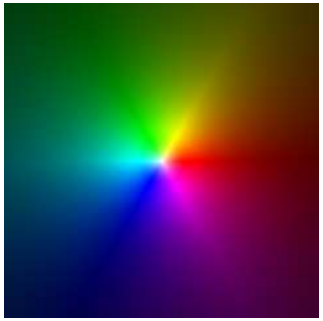




$n = 7$ Ennyi.

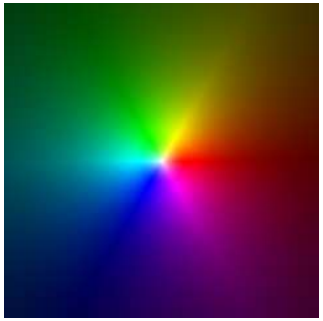
Még egy függvénysorozat

- $$M_0(z) = z$$
  $$M_{n+1}(z) = (M_n(z))^2 + z$$



Még egy függvénysorozat

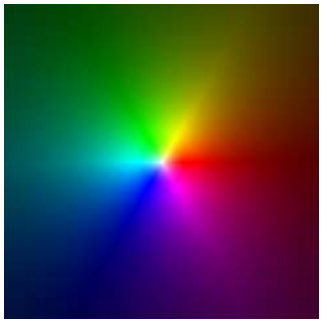
- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$

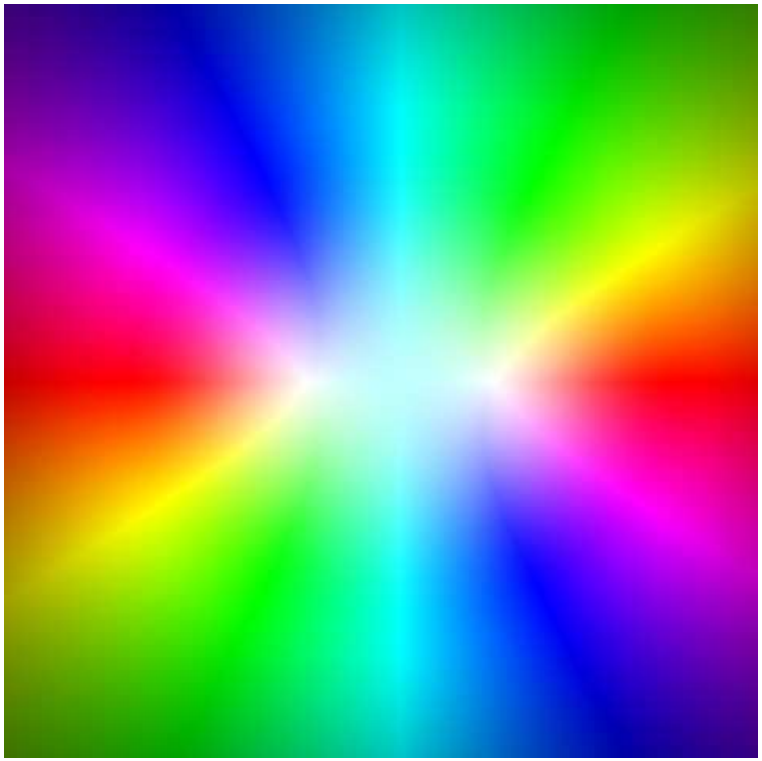


$n = 0$

Még egy függvénysorozat

- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$

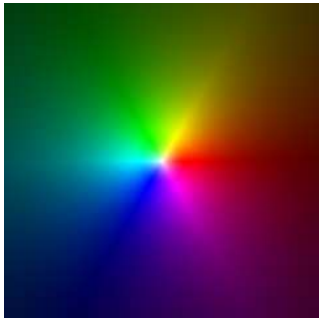


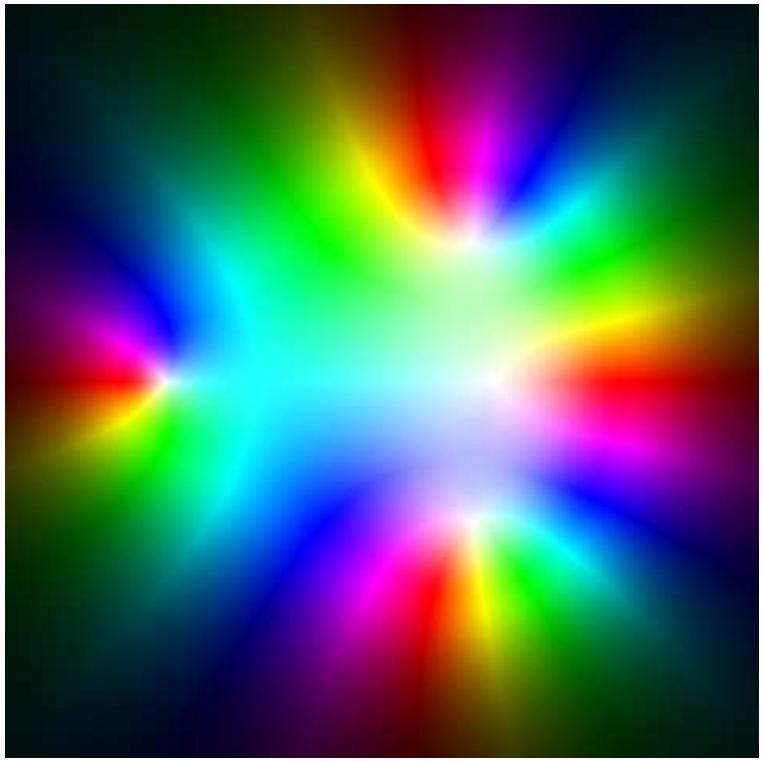


$n = 1$

Még egy függvénysorozat

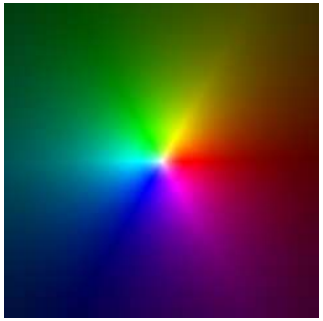
- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$



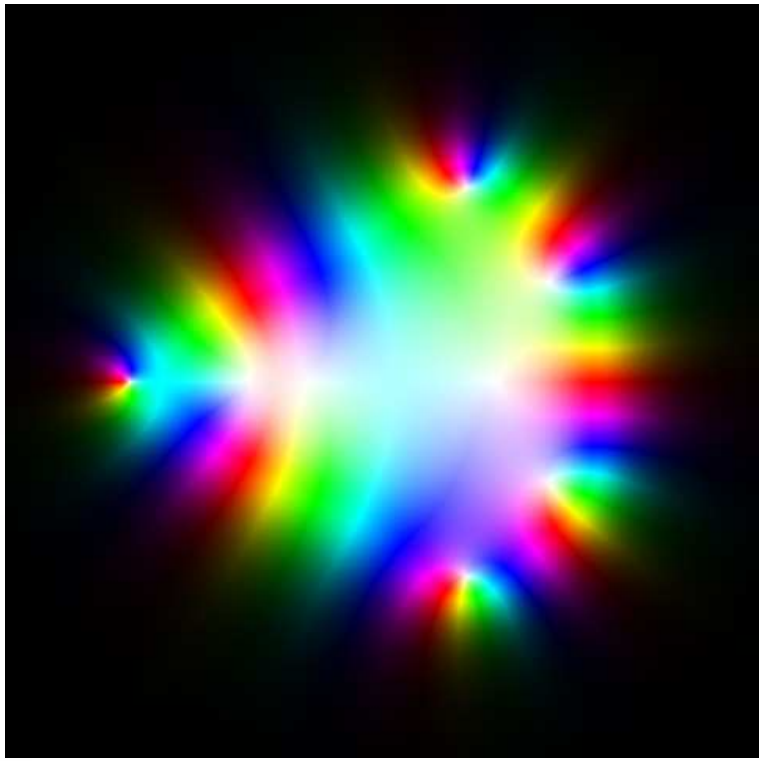


$n = 2$

Még egy függvénysorozat



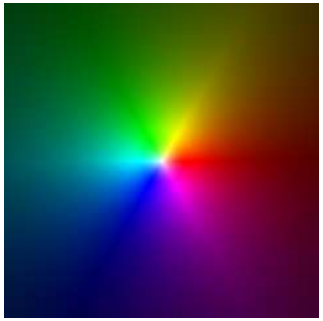
- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$

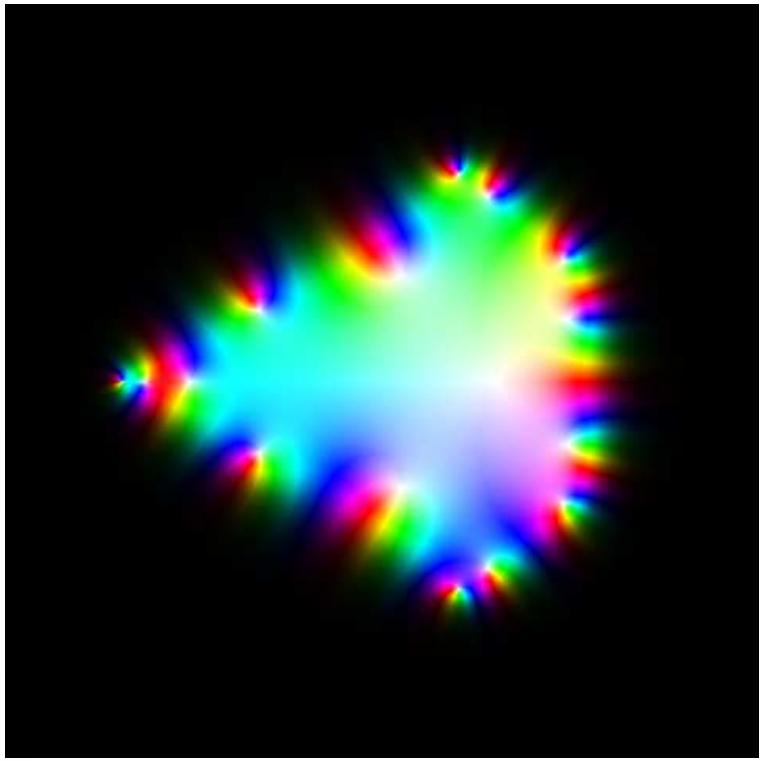


$n = 3$

Még egy függvénysorozat

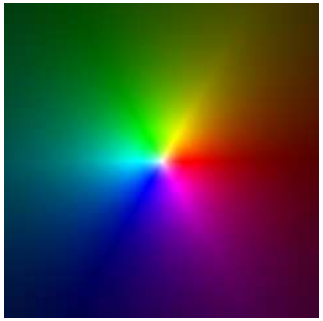
- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$



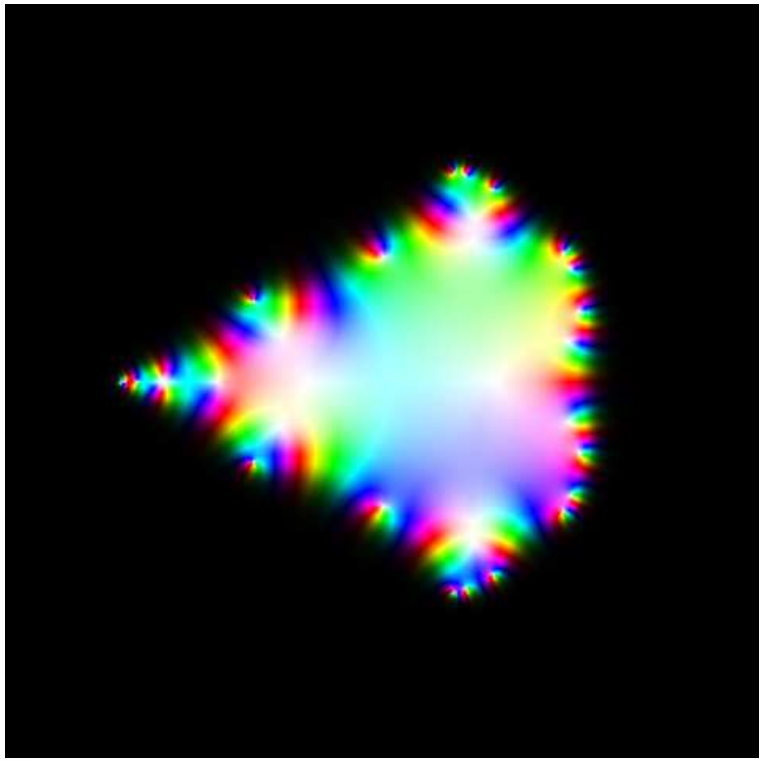


$n = 4$

Még egy függvénysorozat

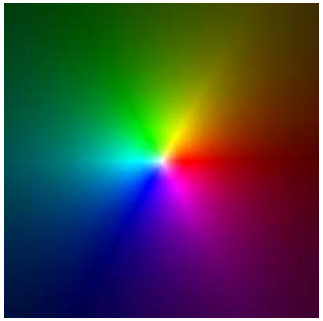


- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$

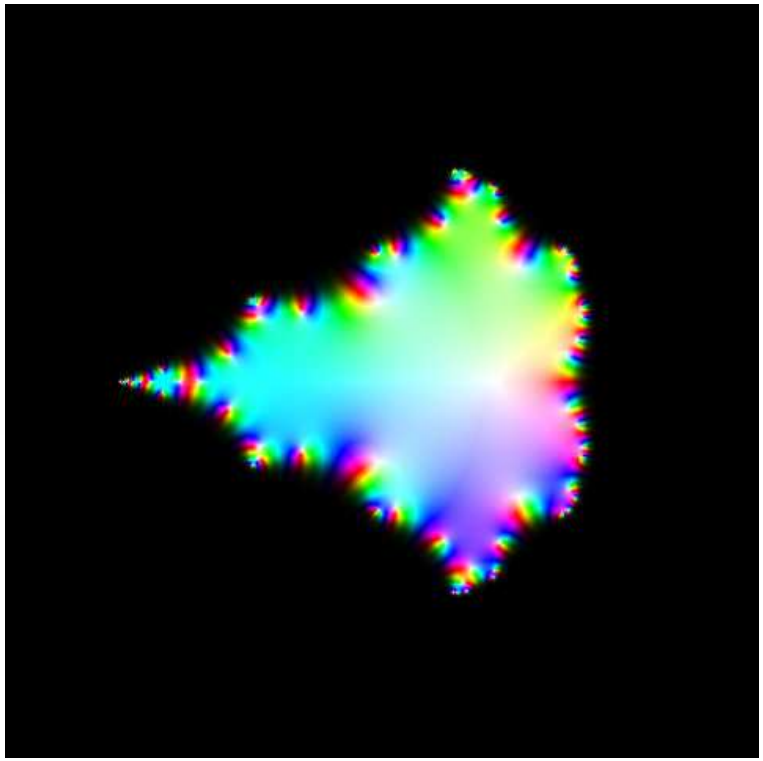


$n = 5$

Még egy függvénysorozat

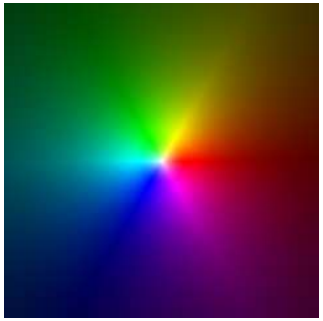


- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$



$n = 6$

Még egy függvénysorozat



- $M_0(z) = z$
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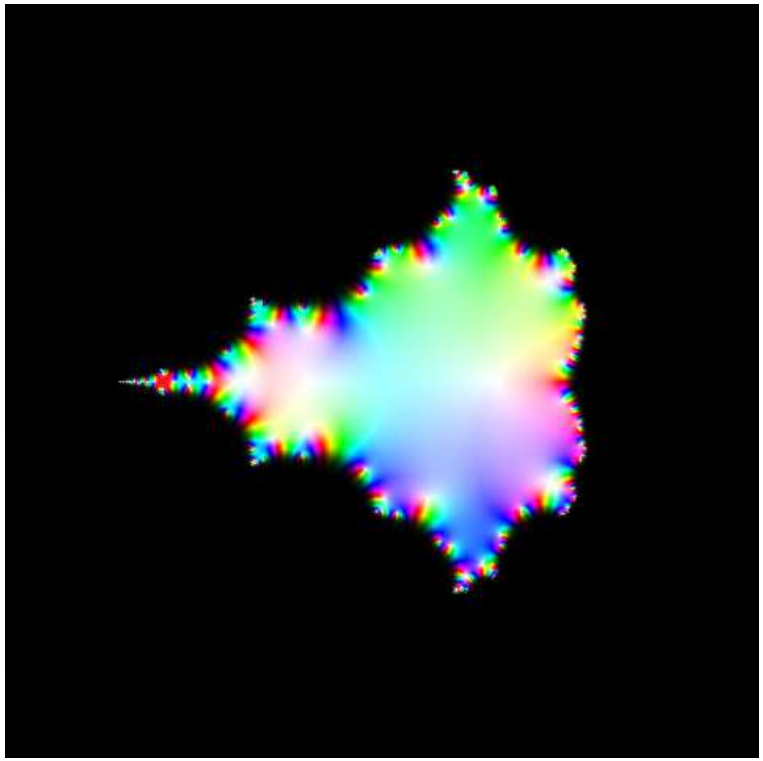
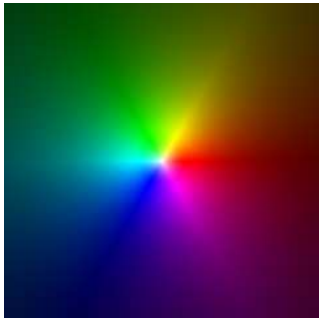
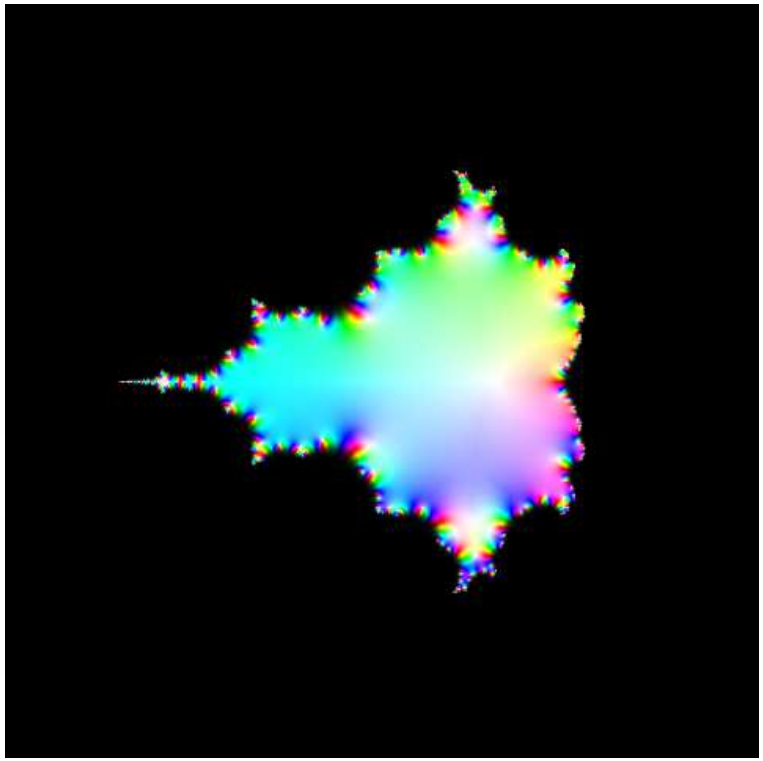


$n = 7$

Még egy függvénysorozat

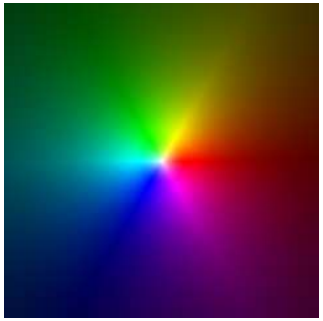


- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$

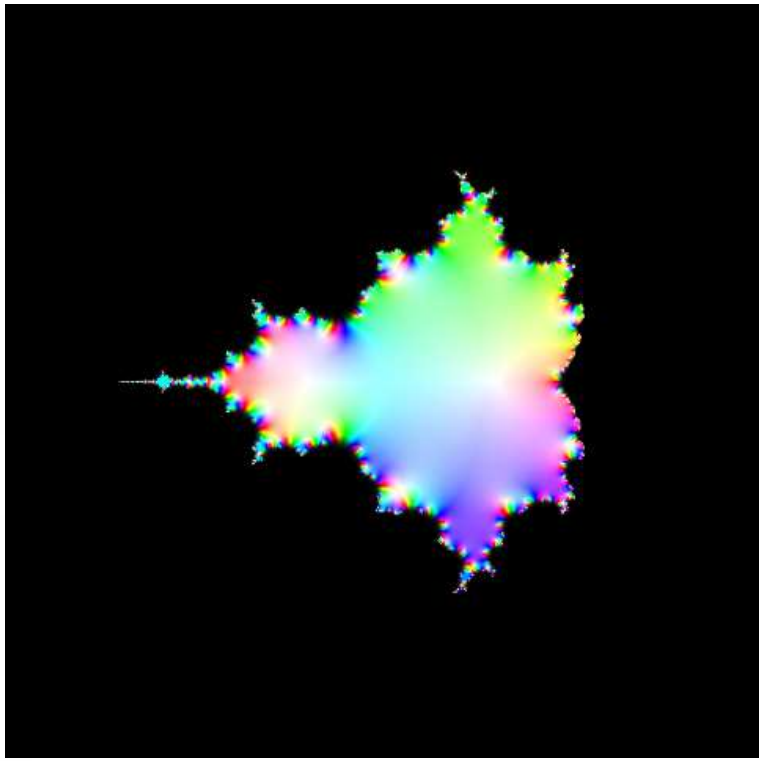


$n = 8$

Még egy függvénysorozat

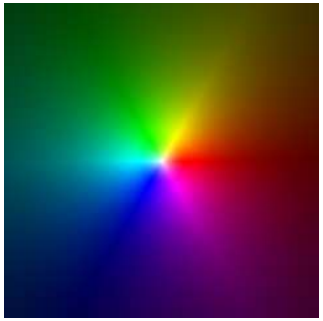


- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$

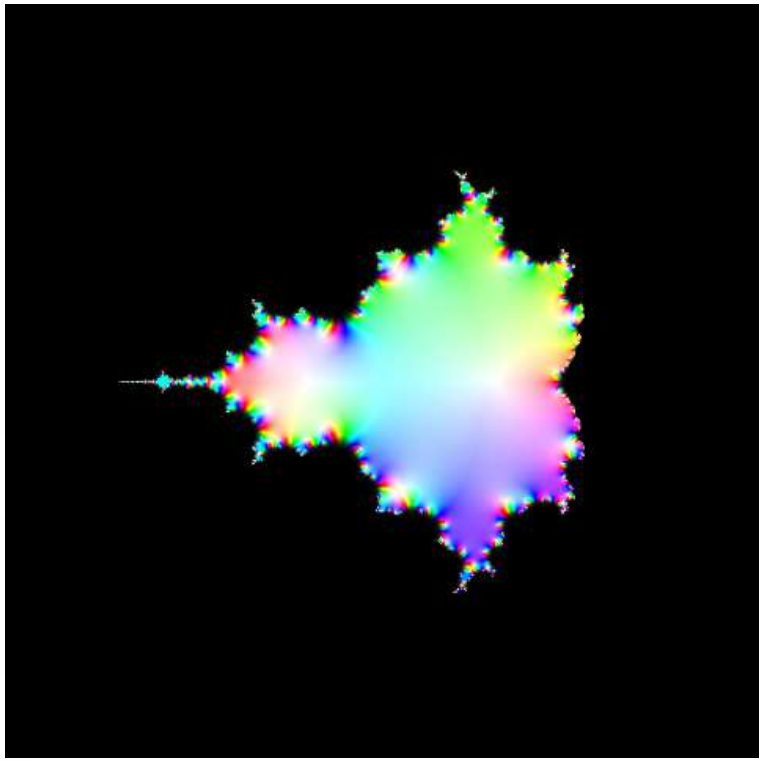


$n = 9$

Még egy függvénysorozat



- $M_0(z) = z$
  $M_{n+1}(z) = (M_n(z))^2 + z$



$n = 9$ Ennyi.

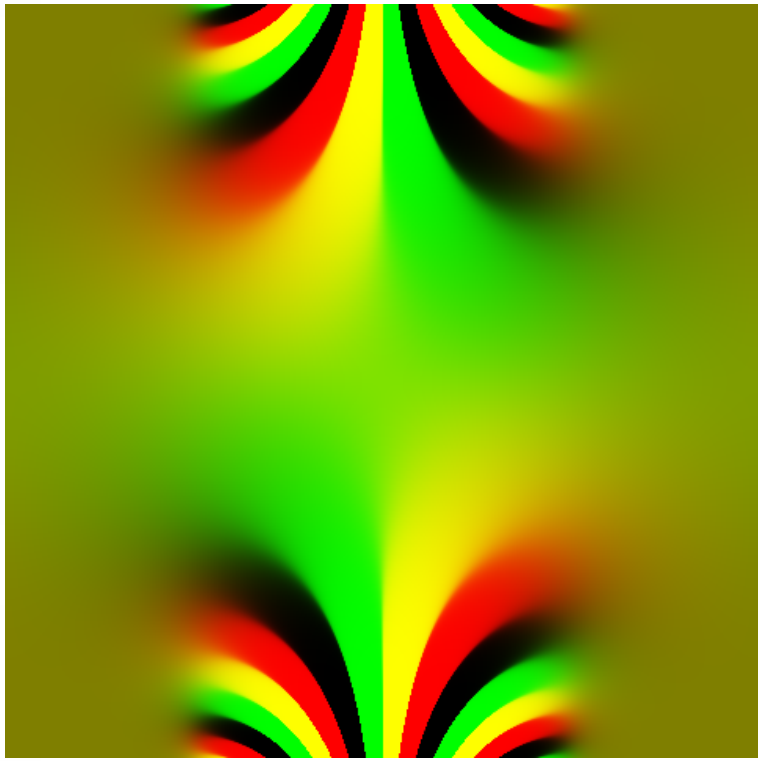
Összefoglalás

- Módszer
- Programozás
- Mi új?

Kedvenc képeim

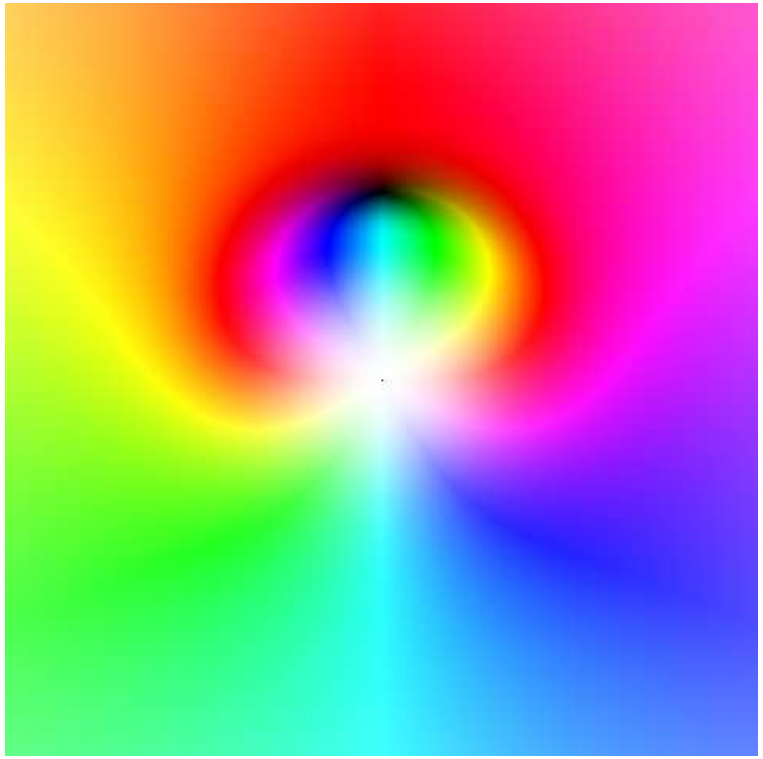
Kedvenc képeim – 1

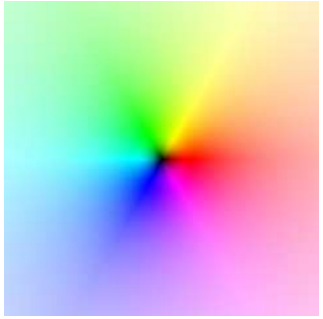
- $f(z) = \exp \sin z$



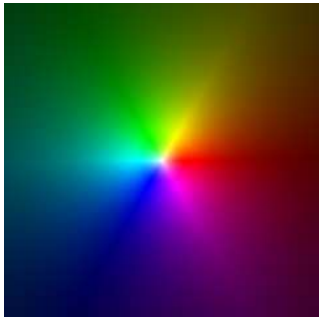
Kedvenc képeim – 2

- $f(z) = \frac{1}{\overline{z}^2} - iz$

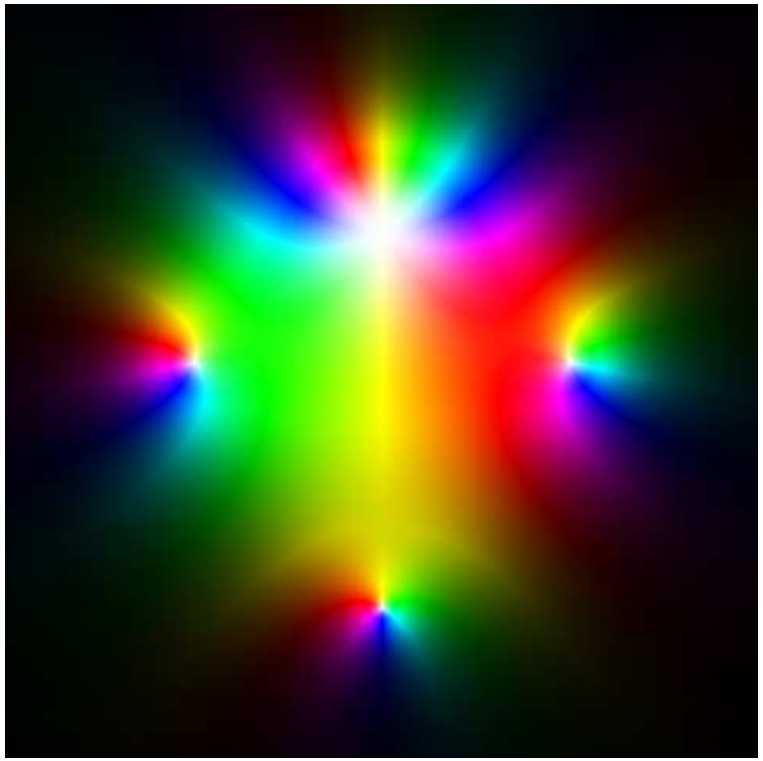




Kedvenc képeim – 3



- Egy ötödfokú polinom



Búcsú

Köszönöm megtisztelő figyelmüket!

Már vége

Készítette:	Lócsi Levente
Alkalom:	TDK Konferencia (2006.12.07.)
Köszönet:	Szili László
	Schipp Ferenc
	Sztupák Sz. Zsolt
	...

Élőben:	Eötvös József Collegium - 322. szoba
Weben:	http://locsi.web.elte.hu/complex
E-mail:	locsi@inf.elte.hu